Opinion Diffusion in Similarity-Driven Networks

Edoardo Baccini[1], Zoé Christoff[1], and Rineke Verbrugge[1]

[1] *University of Groningen*

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"Threshold models" are a widespread approach to modelling diffusion processes on networks. Starting from a set of initial adopters, agents adopt a behavior or an opinion if a big enough proportion of their neighbours has already adopted it [9, 6, 7].

A number of frameworks to reason about diffusion in networks has been proposed, e.g. [1, 4]. Recently, threshold-based approaches have also been used to investigate similarity-driven network changes [13, 15, 12]. The idea is that similarity of behaviors or opinions between agents is one of the mechanisms that drives network change [7, 11]. Agents that are similar enough, e.g., that have enough opinions in common, are more likely to be or become connected, while dissimilar agents are more likely to be or become disconnected.

The main contributions of this presentation are the following: (i) a complete and sound axiom system to reason about synchronous opinion diffusion and similarity-driven network change and (ii) a characterization of their stabilization condition.

Target dynamics. The models that we consider consist of: a social network, which is a graph where nodes represent agents and directed edges represent influence relations; a set of issues and a function that determines, for each agent, the issues that the agent accepts; two rational numbers representing the *influence threshold* for adopting an opinion and the *similarity threshold* for connecting to a similar-enough agent, respectively.

We focus on the following dynamics. First, agents accept an issue if and only if one of the following holds: (i) the influence threshold is 0; (ii) they have no influencers and they already accept it; (iii) a proportion of influencers bigger or equal than the *influence threshold* accept the same issue. Second, as for the network change, one agent influences another if and only if the proportion of issues they agree on meets the *similarity threshold.* We are interested in these dynamics happening simultaneously in a model (an example in Fig. 1).

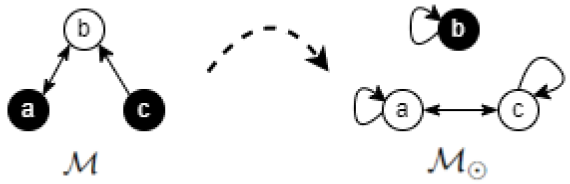

Figure 1: Model update from the initial model $\mathcal{M}$, with influence threshold $\frac{1}{2}$ and similarity threshold 1, to the updated model $\mathcal{M}_\odot$, for the case of a single issue. Black nodes represent agents that accept the issue, and white nodes represent the agents that reject it. Directed arrows represent the influence of an agent on another. The dotted arrow stands for the update transition.

A dynamic logic for synchronous change. We make two strong assumptions with respect to the thresholds: they are uniform across the agents and they do not change. Similarly to [13], we develop a dynamic propositional logic to reason about model changes. We start by defining a syntax and semantics to talk about the agents' opinions and their influence relations. We then enrich the static syntax with a dynamic operator that captures the simultaneous changes in the network and in the agents' opinions.

We then give an axiomatization, and show its soundness and completeness. Completeness is shown by reducing the logic to its static fragment, as is typically done in Dynamic Epistemic Logic [10, 5].

Stability and stabilization As done in [8], we restrict our attention to the case of a single binary issue. We start by showing that after one update, every model is symmetric and reflexive. Building on this, we show that a stable model can either be: (i) a network where every agent is connected to every other agent, and in which all agents have the same opinion; (ii) a network consisting of two complete and mutually disjoint components, where all agents in one component accept the target issue and all agents in the other reject it.

As in other related literature, e.g. [3, 8], our main interest is the stabilization condition of iterated sequences of model-updates. In this regard, we provide necessary and sufficient conditions for a given initial model to stabilize.

We do that by first defining a partition of the set of agents of any initial model into four sets: agents that accept the single issue at hand and have enough pressure to accept it; agents that accept the single issue at hand but do not have enough pressure to accept it; agents that reject the single issue at hand but have enough pressure to accept it; agents that reject the single issue at hand and do not have enough pressure to accept it. We provide a characterization of the models that stabilize in terms of the comparative sizes of these four sets. Furthermore, prompted by the work in [2], we also analyse the oscillatory behavior of models that do not stabilize by characterizing the length of their oscillation.

Generalizations Two generalizations of our framework are discussed. First, we extend our analysis of stability to the case with multiple issues, which leads to the proof of a number of properties characterizing the structure of social networks in stable models.

Second, we extend the logic so as to also account for changes in the network and in the opinion diffusion happening at *different* times. For that, we draw from the work in [14], and define two distinct model updates, one for the network change and one for the opinion diffusion, respectively. Accordingly, the logical syntax of the dynamic logic is enriched with two distinct dynamic operators for network change and opinion diffusion, respectively.

A new axiom system, combining synchronic and diachronic operators is provided and shown to be sound and complete via reduction axioms. Furthermore, we point out a number of interesting questions that arise regarding the expressivity of synchronic and diachronic operators.

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