Comparing Social Network Dynamics

Edoardo Baccini[1] joint work with Zoé Christoff[1] and Rineke Verbrugge[1]

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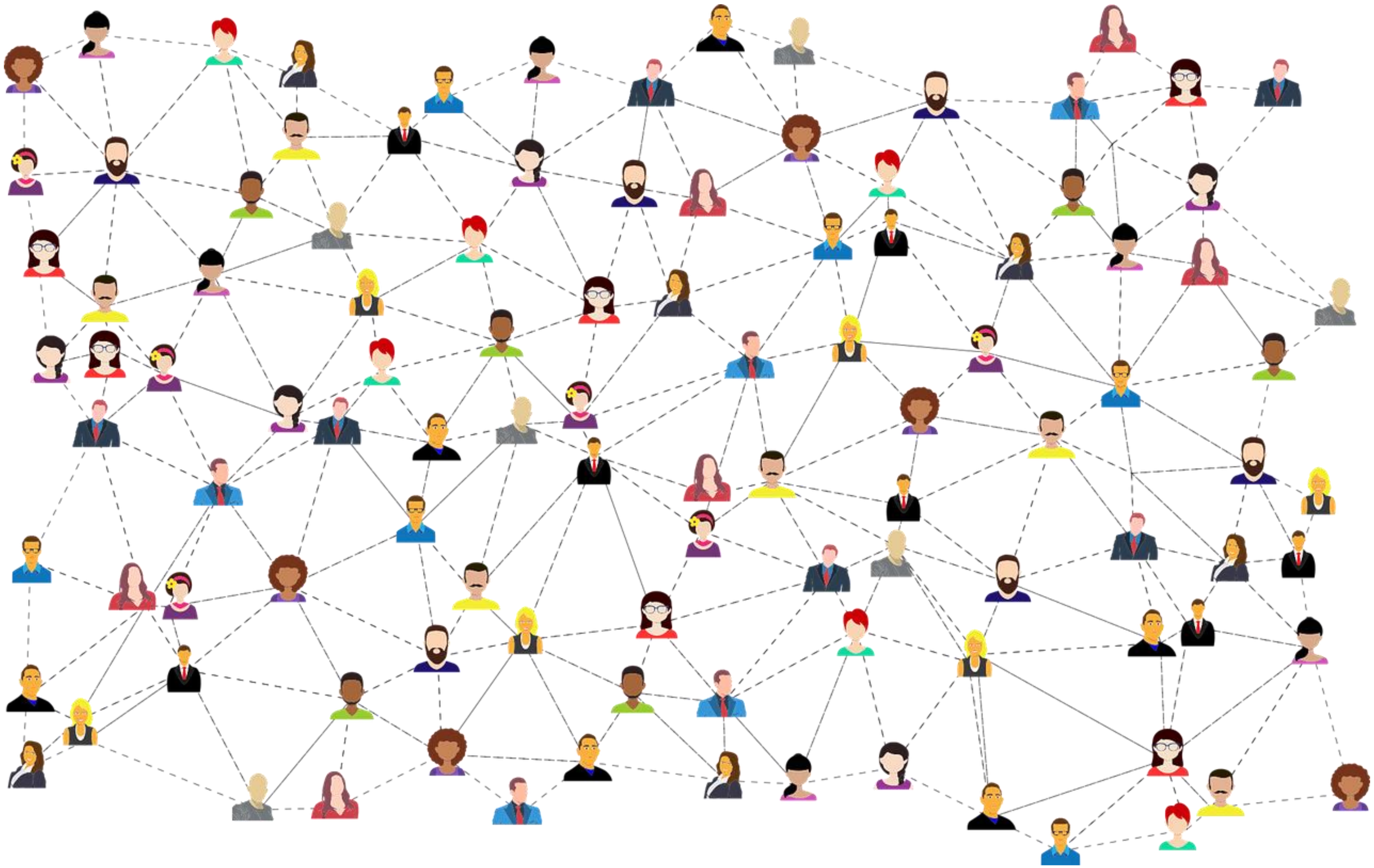
Social Networks



Social Network Dynamics

Diffusion Processes
Agents' opinions are influenced by the opinions of their network neighbours.

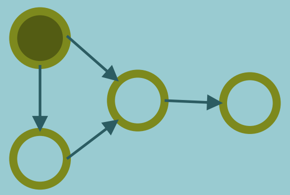
Link Change
Agents connect to similar agents and disconnect from dissimilar agents.

Social Network Dynamics

Threshold Models of Diffusion

-Social Network
-Agents initially possessing a opinion
-Uniform Influenceability threshold

Update Rule
Adopt if **enough** network neighbours have already adopted.



Easly and Kleinberg (2010), Baltag et al. (2019)

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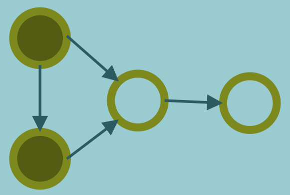

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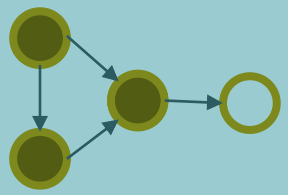

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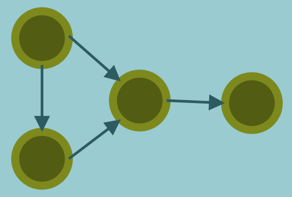

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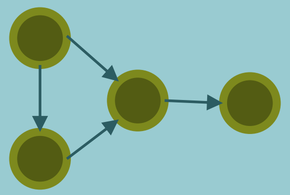
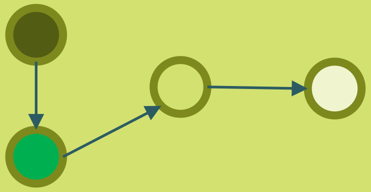


Easly and Kleinberg (2010), Baltag et al. (2019)

Threshold-based Link Change

-Social Network
-Set of opinions
-Uniform similarity threshold

Connection Rule
Connect if you have **enough** features in common.



Smets and Velázquez-Quesada (2019, 2020)

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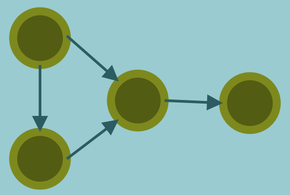

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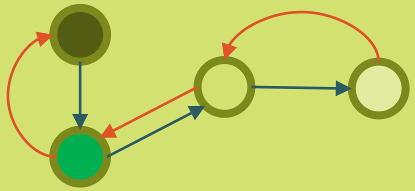

Smets and Velázquez-Quesada (2019, 2020)

Models of Social Network Dynamics

Monotonic
Diffusion

Old opinions stay!

e.g., Easley and Kleinberg (2010), Baltag et al. (2019)

Monotonic
Link Change

Old ties must stay!

e.g., B. and Christoff (2023)

Models of Social Network Dynamics

Monotonic
Diffusion

Old opinions stay!

e.g., Easley and Kleinberg (2010), Baltag et al. (2019)

Non-monotonic
Diffusion

Old opinions can go!

e.g., Smets and Velázquez-Quesada(2019), Grandi et al. (2015)

Monotonic
Link Change

Old ties must stay!

e.g., B. and Christoff (2023)

Non-monotonic
Link Change

Old ties can go!

e.g., Smets and Velázquez-Quesada(2019, 2020)

Models of Social Network Dynamics

Model Analysis

Monotonic Diffusion Old opinions stay!	**Non-monotonic** Diffusion Old opinions can go!
Monotonic Link Change Old ties must stay!	**Non-monotonic** Link Change Old ties can go!

Models of Social Network Dynamics

Model Analysis

Monotonic Diffusion Old opinions stay!	**Non-monotonic** Diffusion Old opinions can go!
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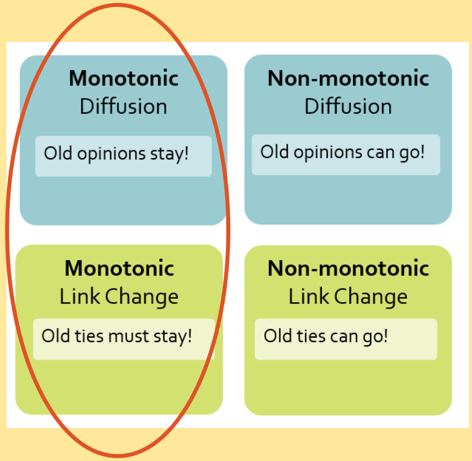
Fix a set of operations

Models of Social Network Dynamics

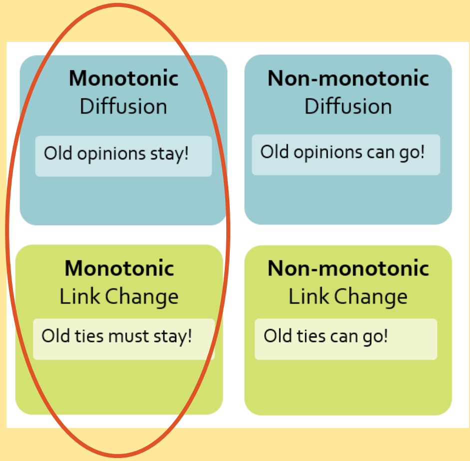
Model Analysis



Fix a set of operations

Models of Social Network Dynamics

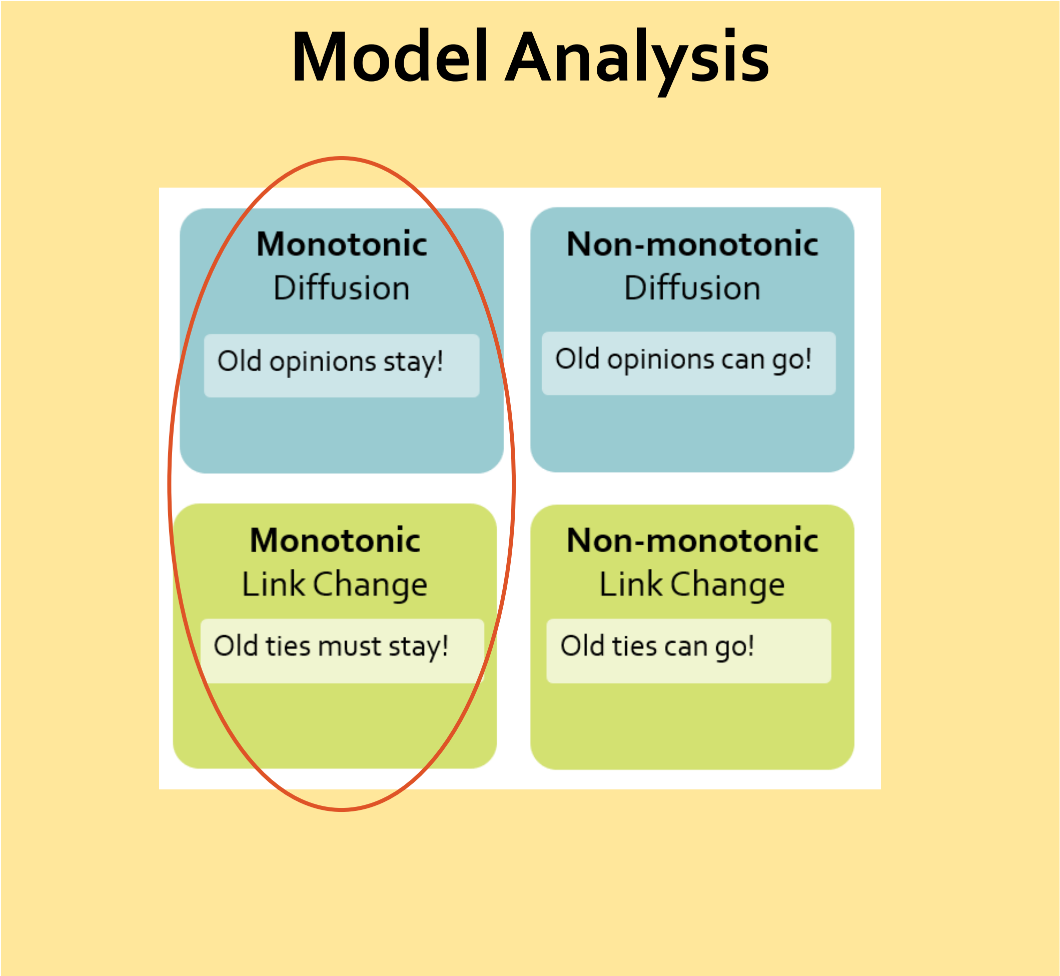
Model Analysis



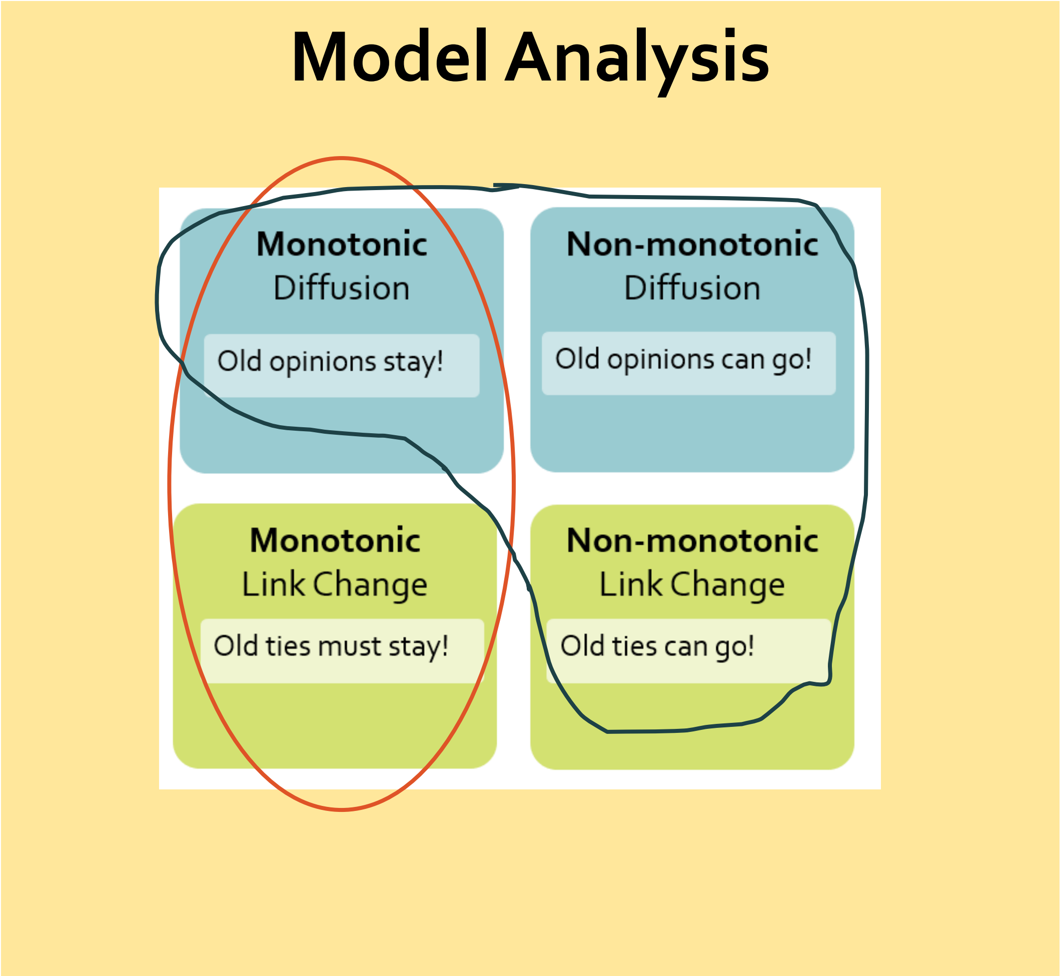
Fix a set of operations

Study **sequences** of operations

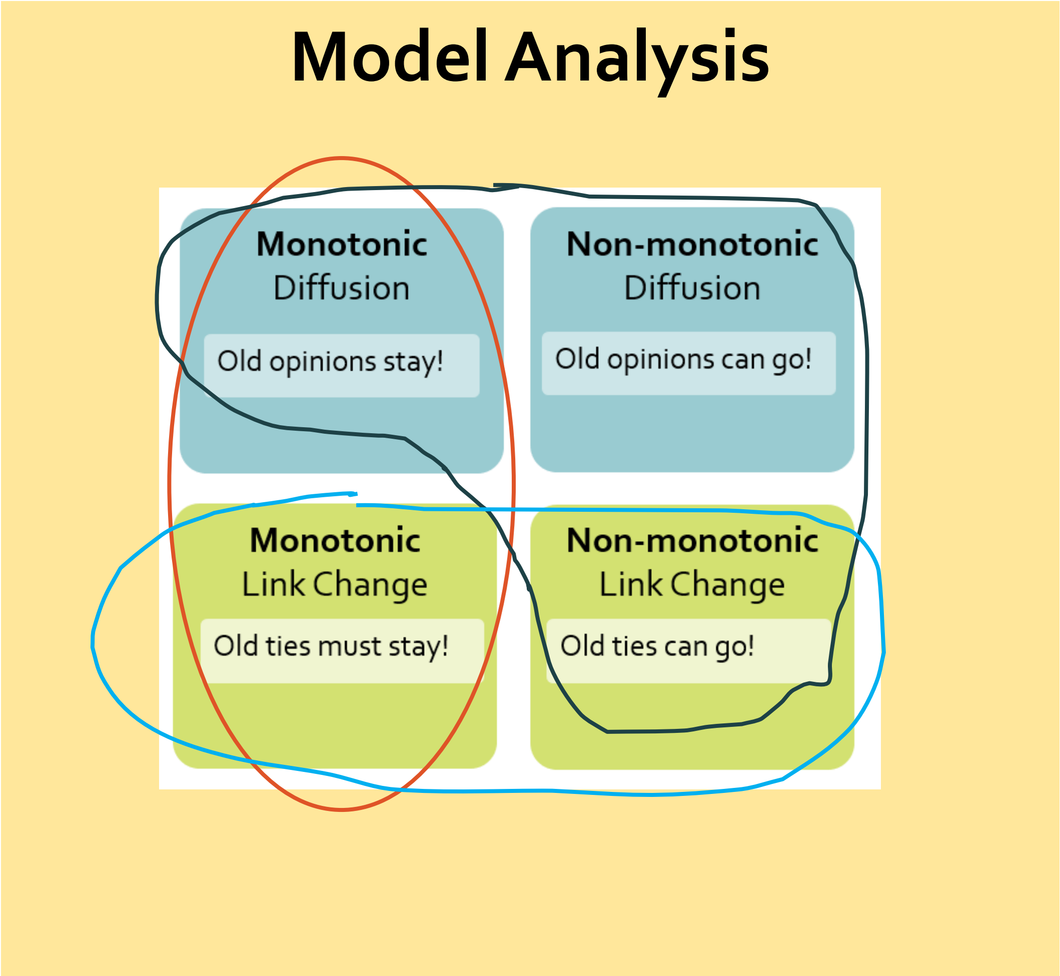
Models of Social Network Dynamics



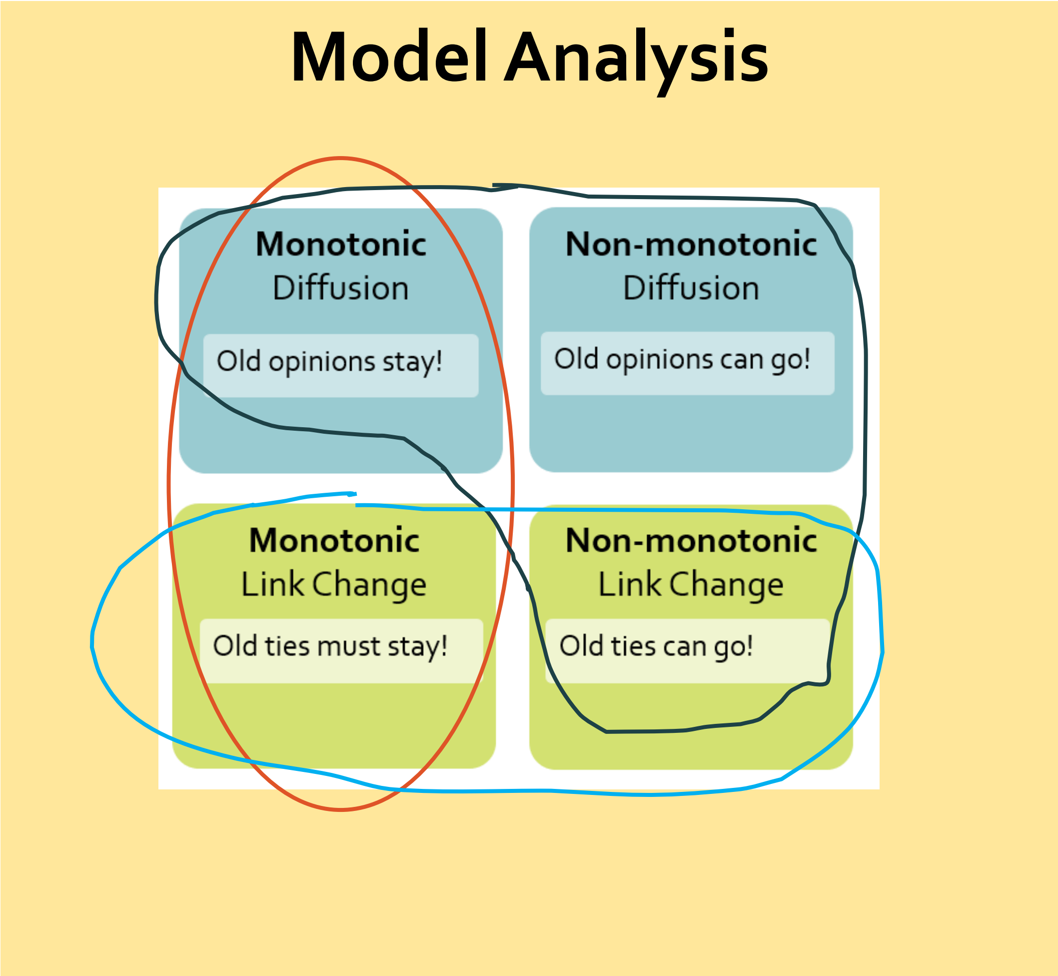
Models of Social Network Dynamics



Models of Social Network Dynamics



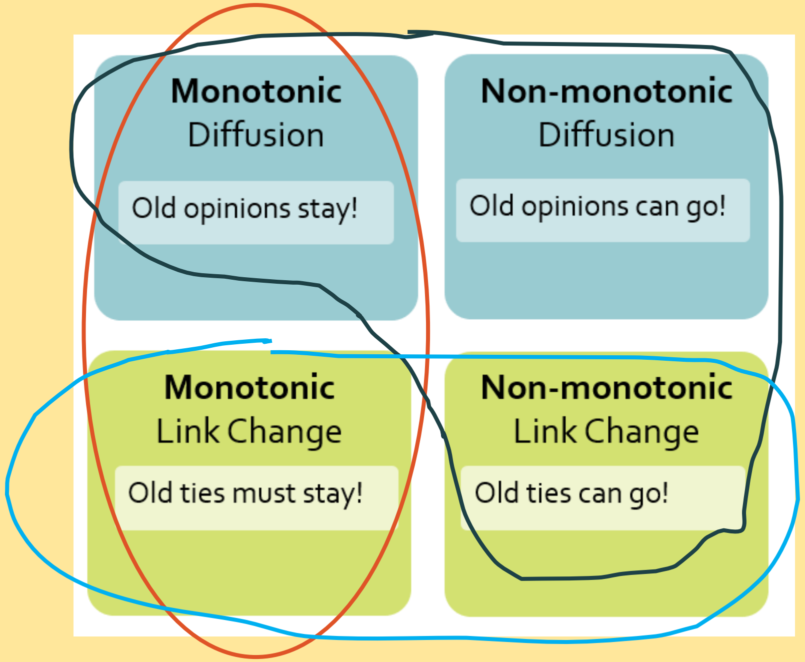
Models of Social Network Dynamics



Q	Do different sets capture different dynamics?

Models of Social Network Dynamics

Model Analysis



Q Do different sets capture different dynamics?

Q Can we reduce sets of operations to others?

Comparing Social Network Dynamics

1. Design a logic for social network changes.

2. Comparing dynamics via replaceability

2.1 General irreplaceability result

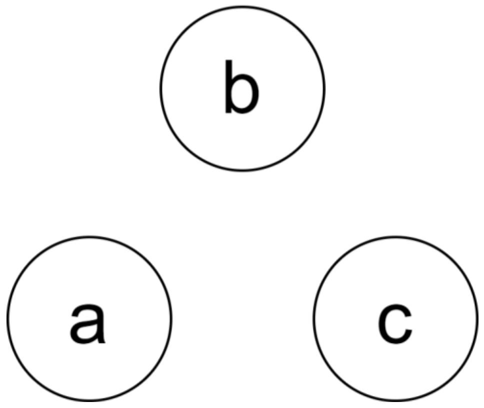
2.2 Replaceability on special classes of models

Comparing Social Network Dynamics

1. Design a logic for social network changes.

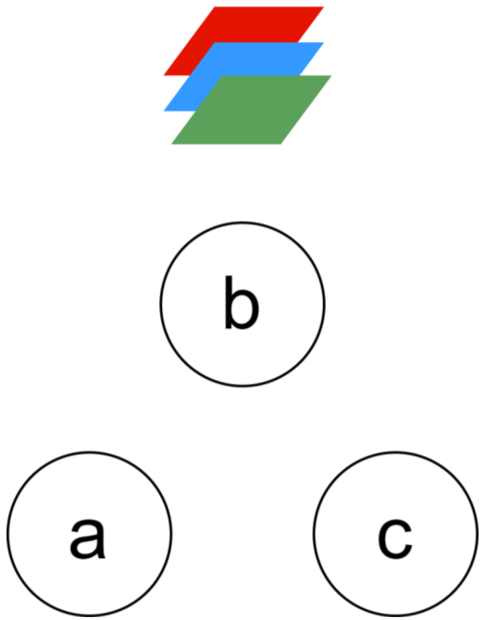
A Logic for Social Network Changes

1. **Set of agents**



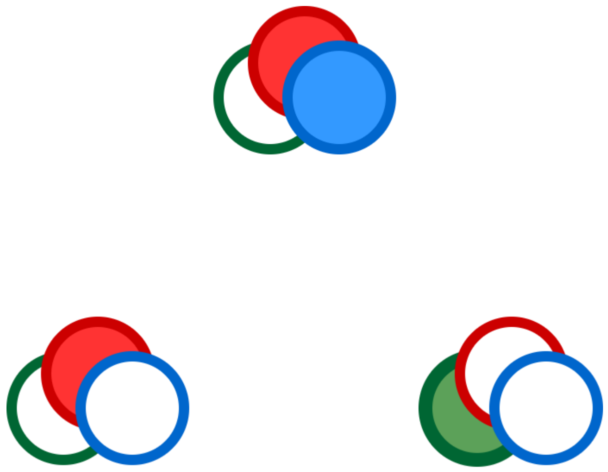
A Logic for Social Network Changes

1. Set of agents
2. Set of features



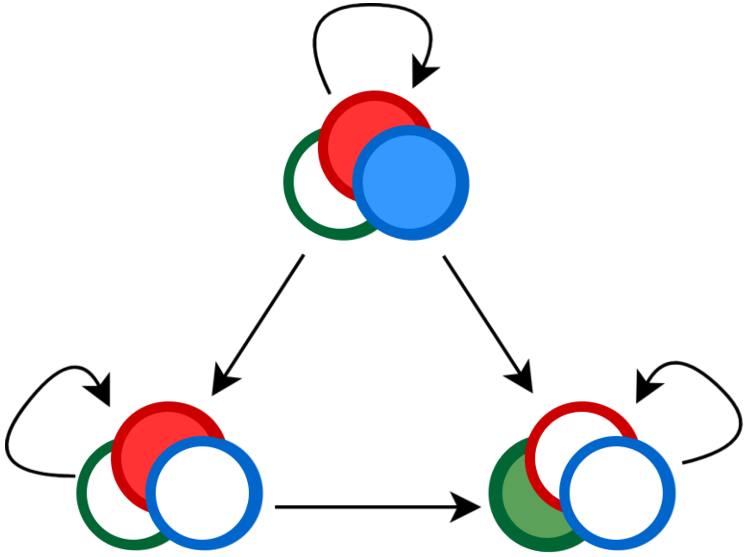
A Logic for Social Network Changes

1. Set of agents
2. Set of features
3. Features assignment



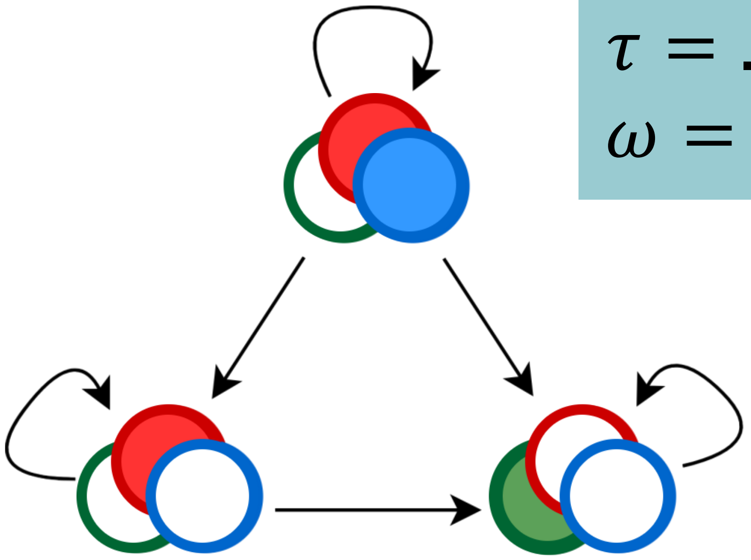
A Logic for Social Network Changes

1. Set of agents
2. Set of features
3. Features assignment
4. Influence Network



A Logic for Social Network Changes

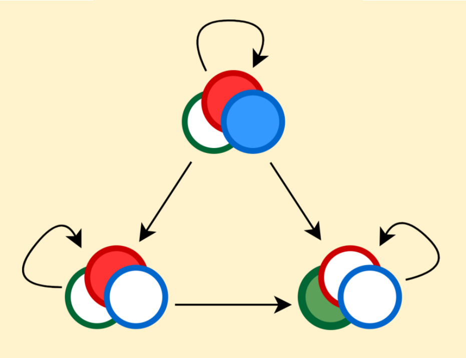
1. Set of agents
2. Set of features
3. Features assignment
4. Influence Network
5. Influenceability threshold $\tau > 0$
6. Similarity threshold $\omega > 0$



$\tau = .5$
$\omega = .3$

Model Updates

Initial Model



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	Diffusion	Link Change	Simultaneous Changes
Monotonic	△ [1]	□ [2]	○ [2]
Non Monotonic	⟁ [3]	⊡ [3]	⊙ [3]

1. See also Baltag et al. (2019).
2. See also B., Christoff and Verbrugge (2022), B. and Christoff (2023)
3. See also Smets and Velázquez-Quesada(2019, 2020).

Model Updates

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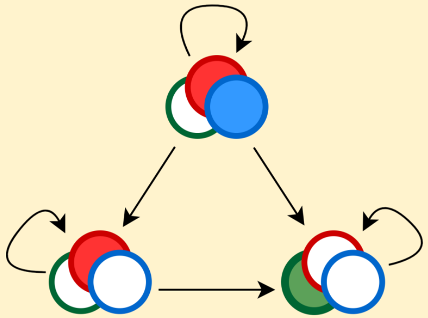

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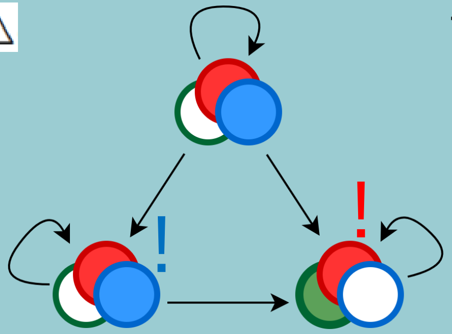
Monotonic	△ 1 	□ 2	○ 2
Non Monotonic	△ 3	⊡ 3	⊙ 3

1. See also Baltag et al. (2019).
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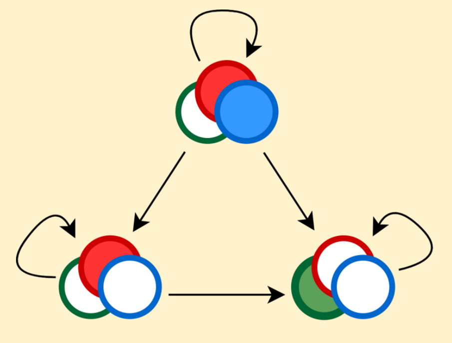

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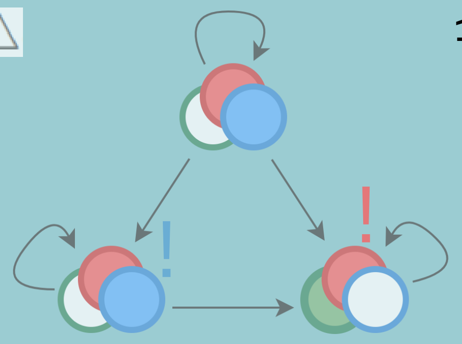
Monotonic	△ [1] 	□ [2]	○ [2]
Non Monotonic	◬ [3]	⊡ [3]	⊙ [3]

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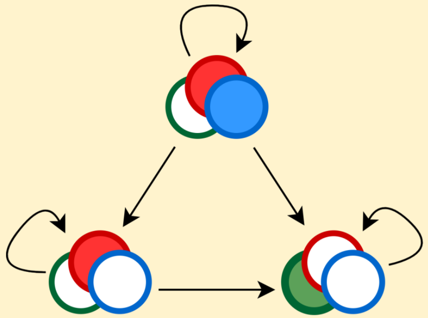

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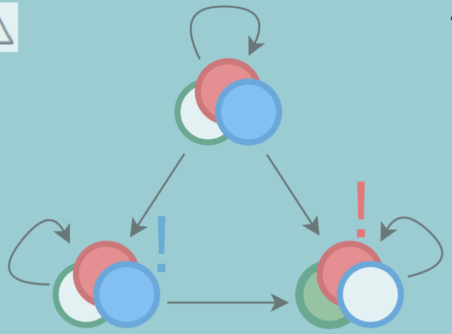
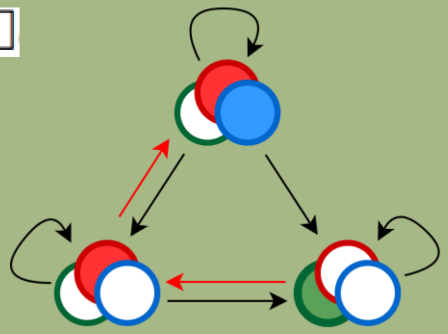
Monotonic	△ 1 	□ 2 	○ 2
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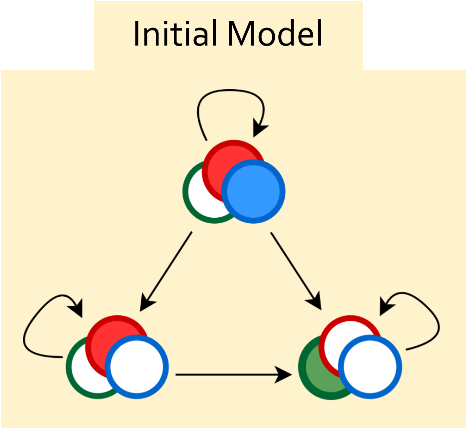

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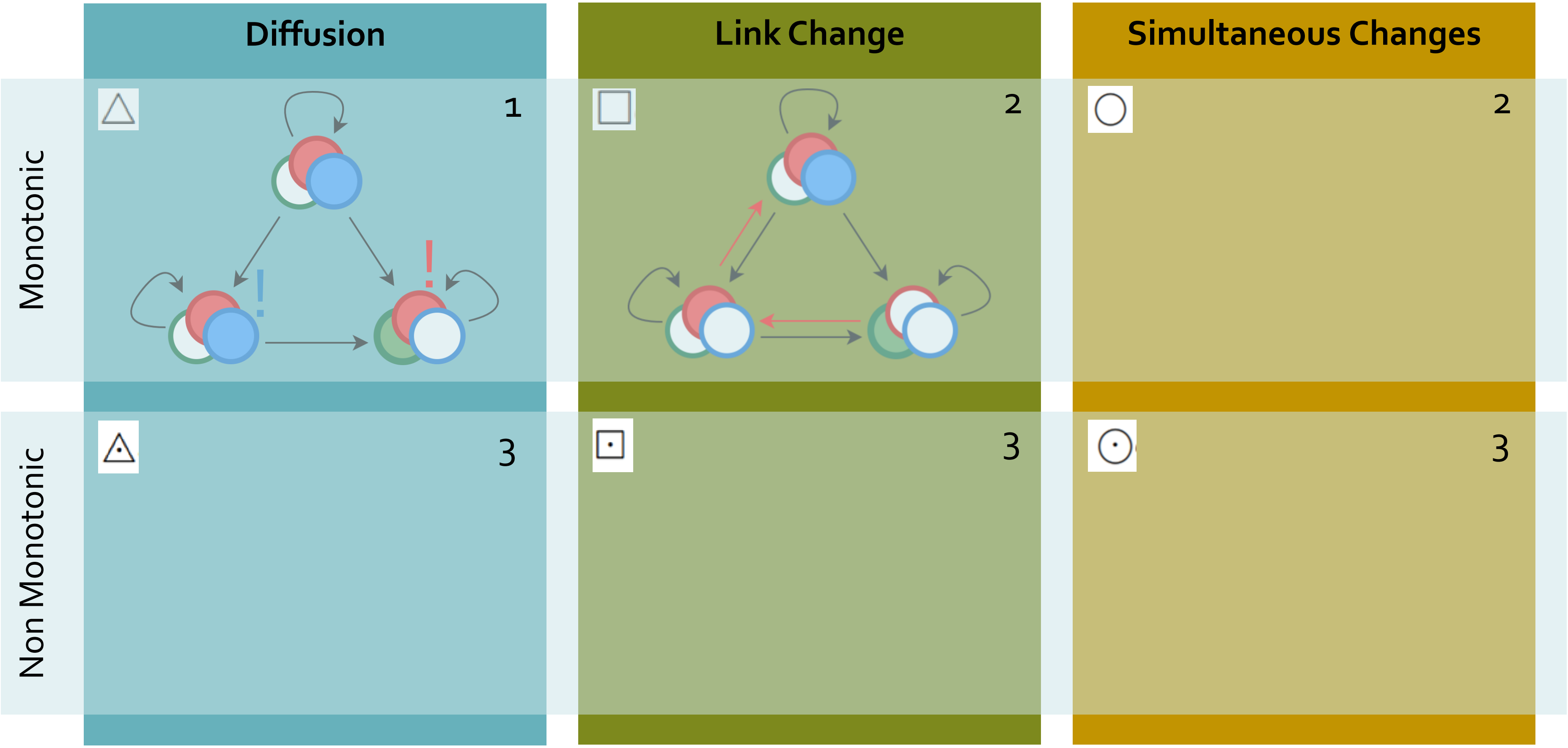

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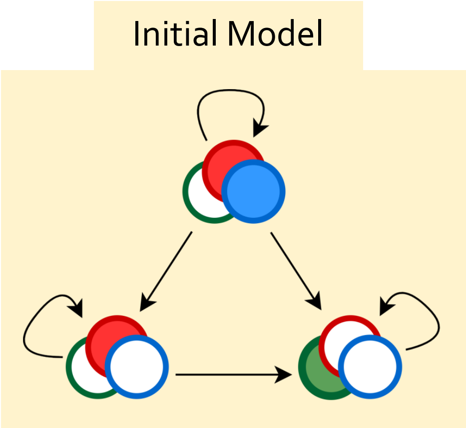

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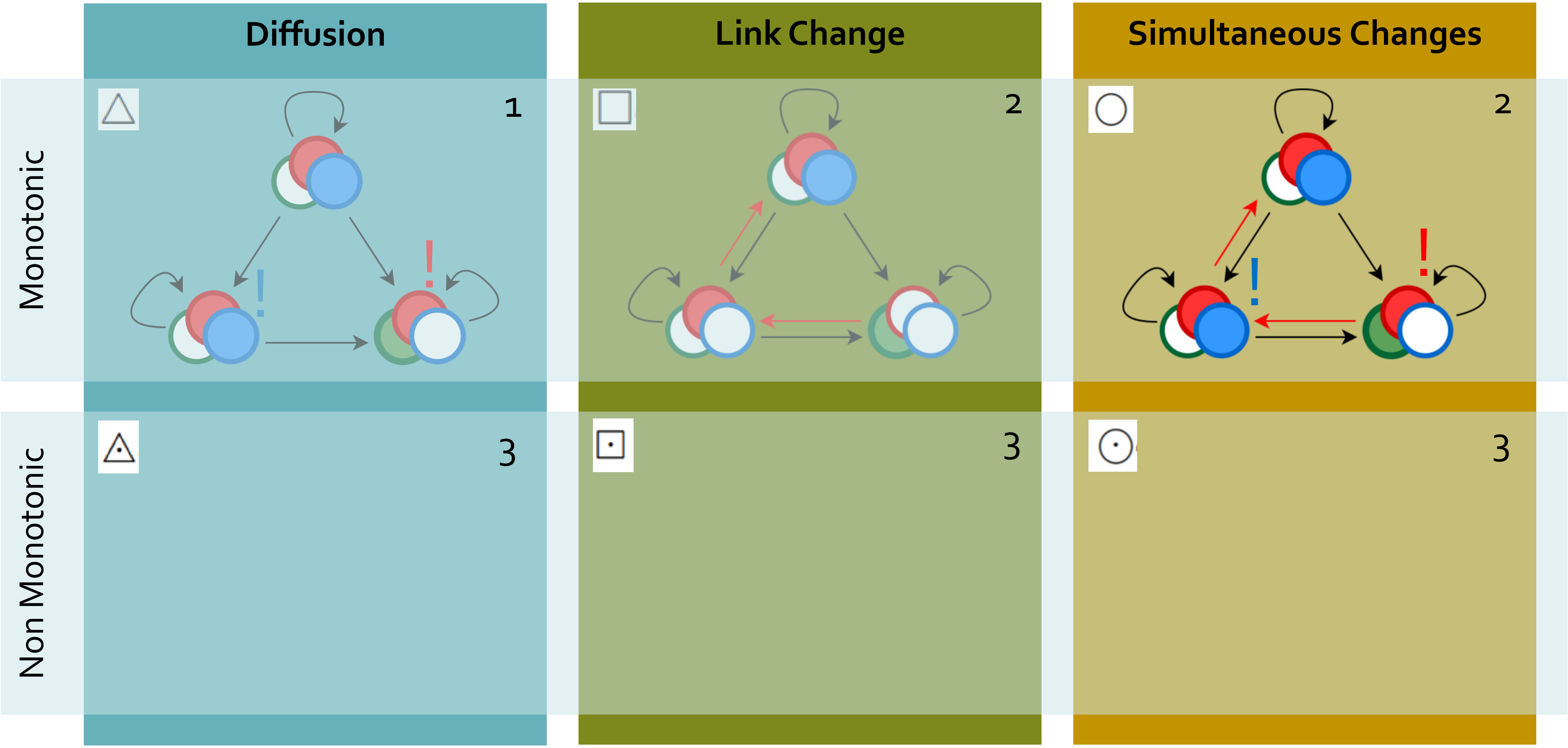

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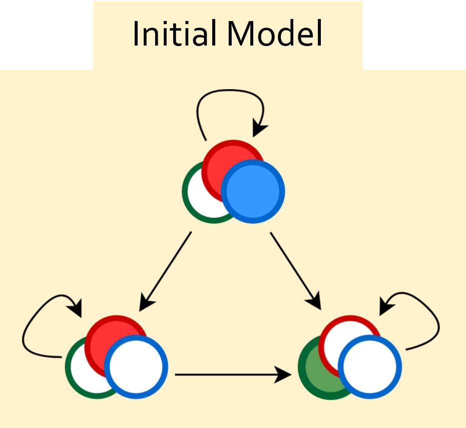

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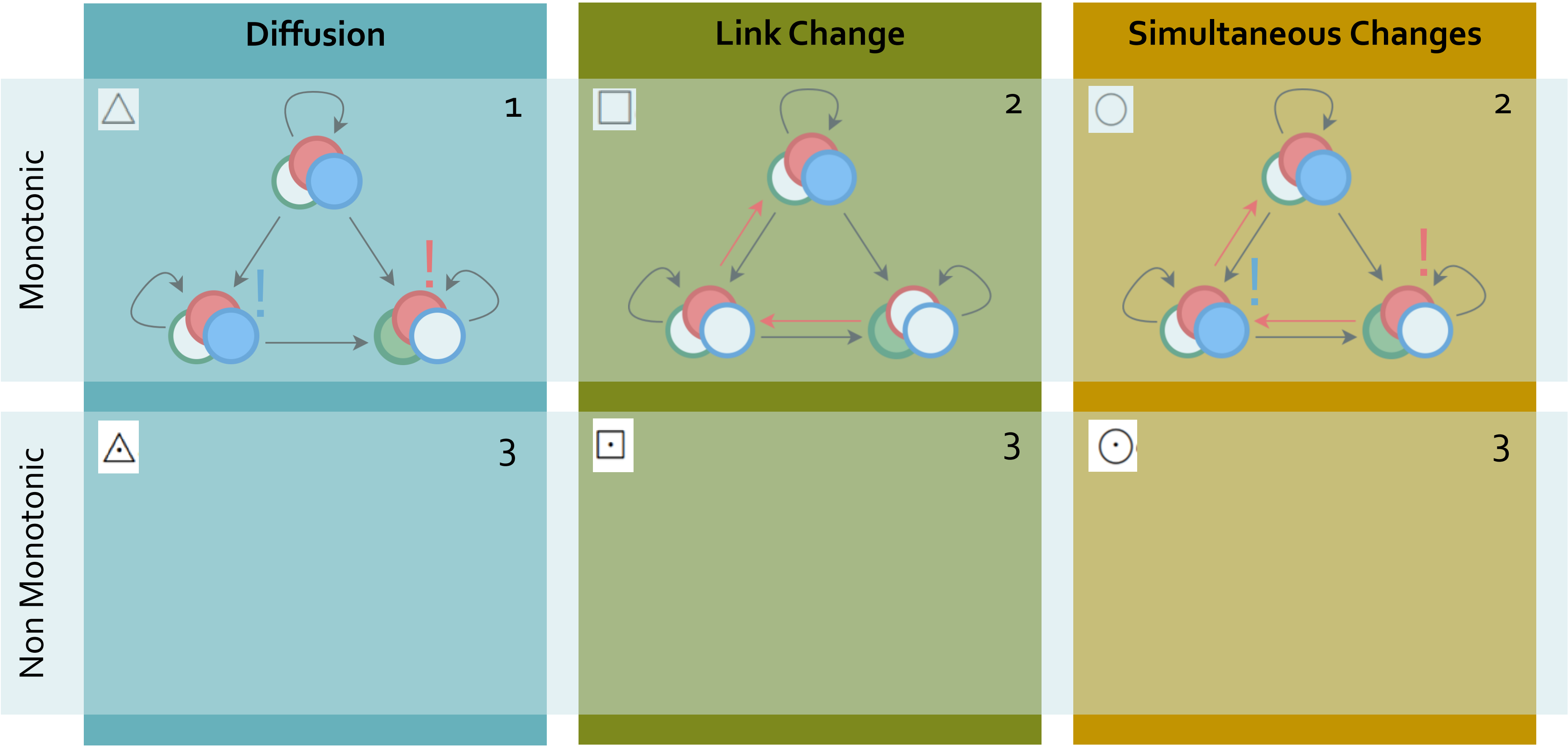

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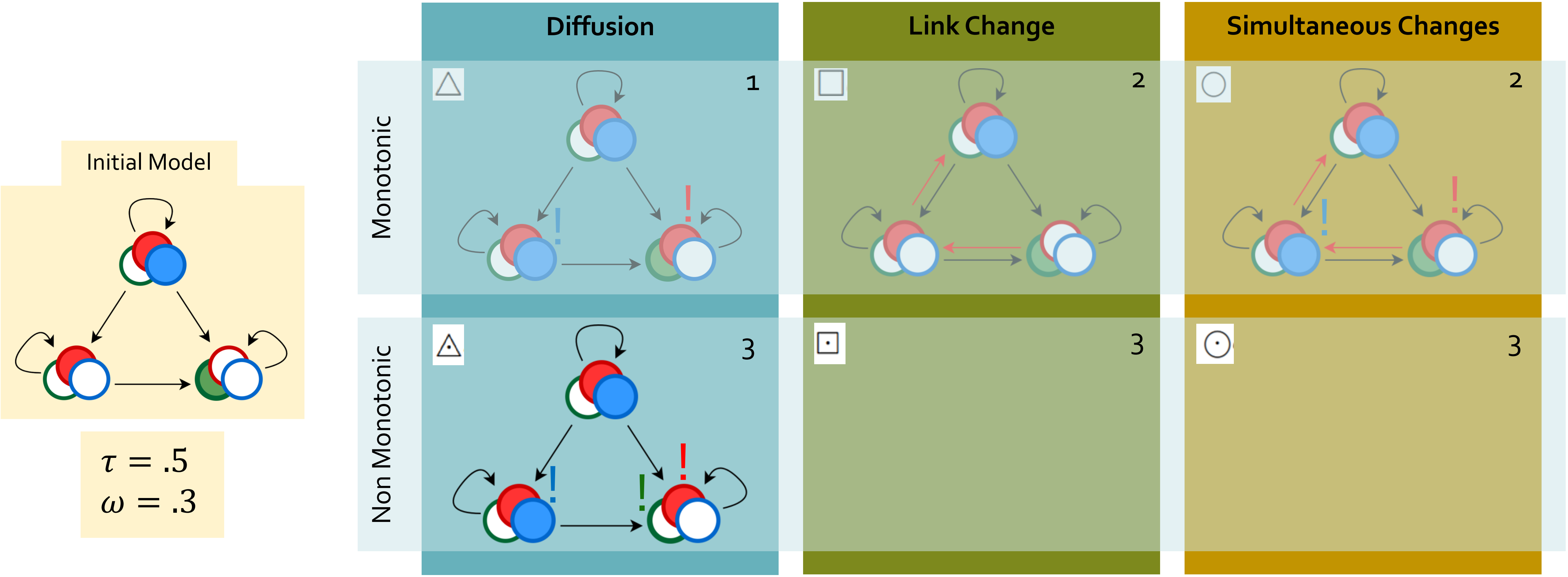

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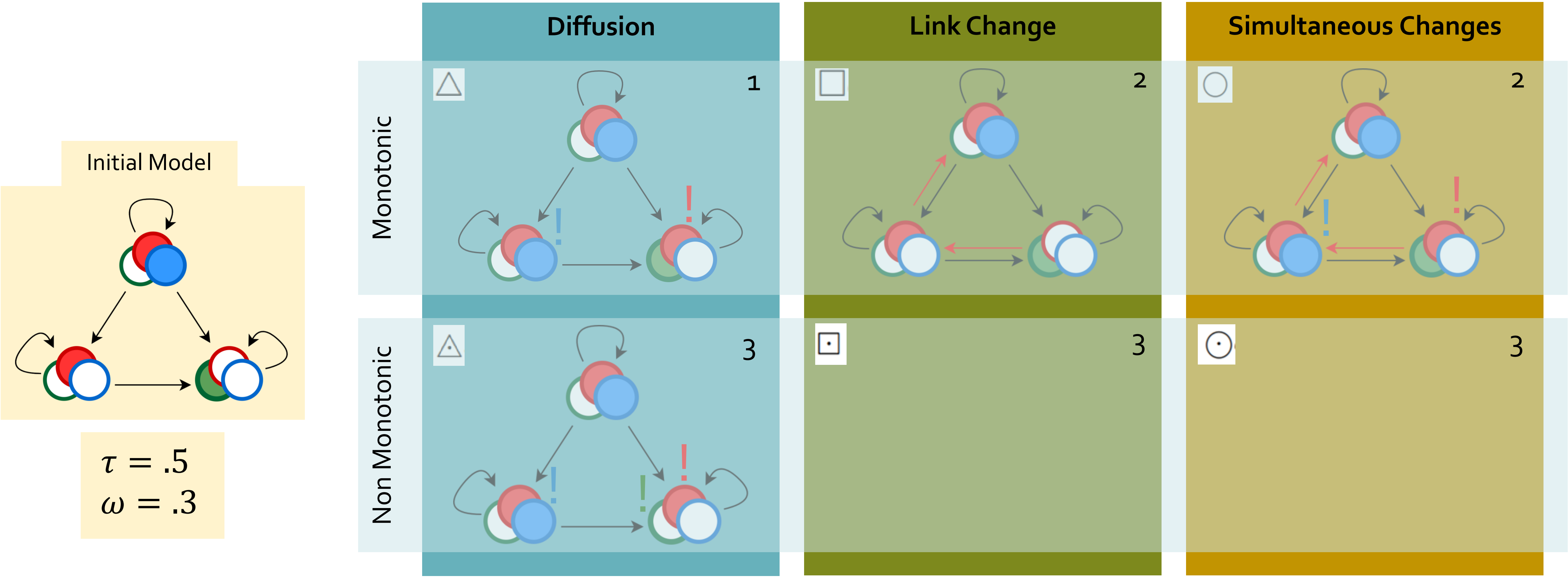

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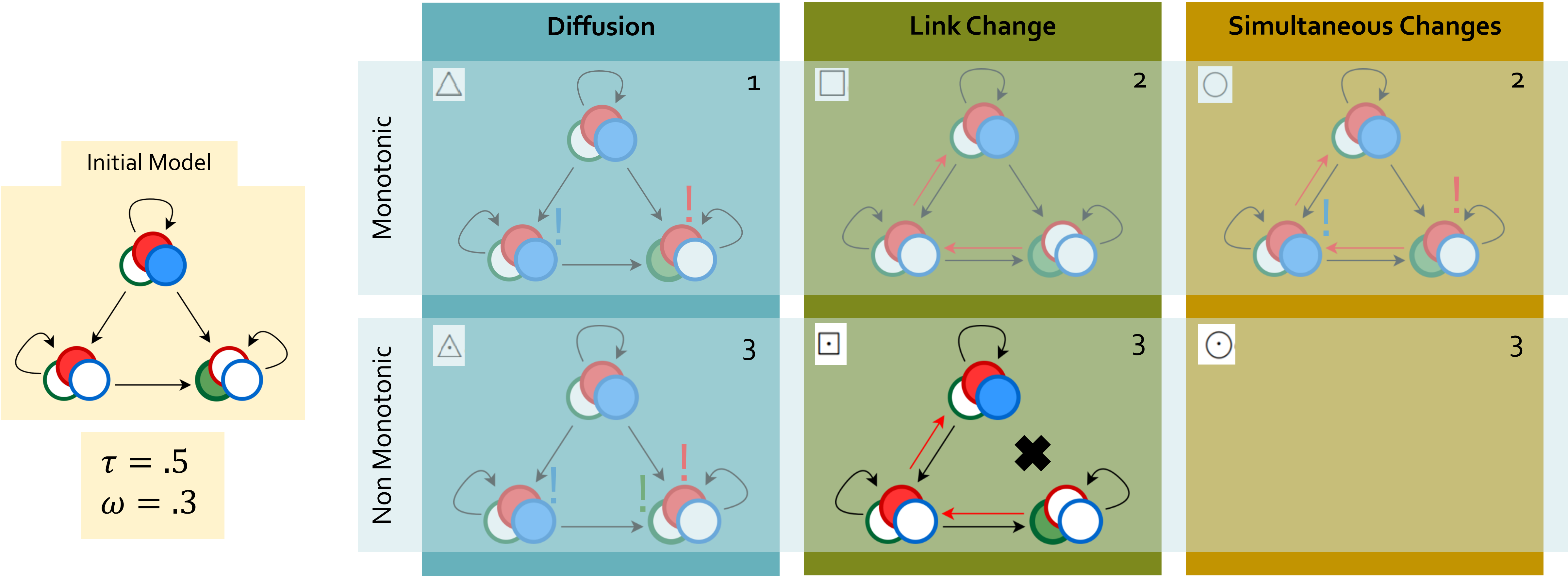

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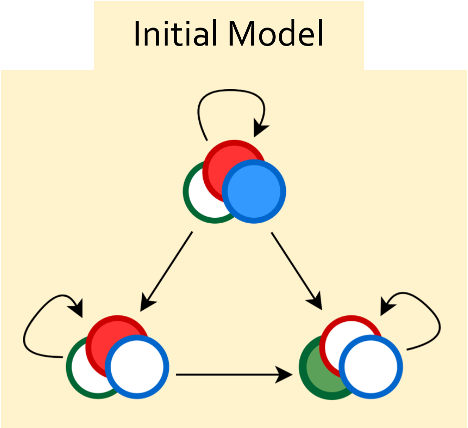

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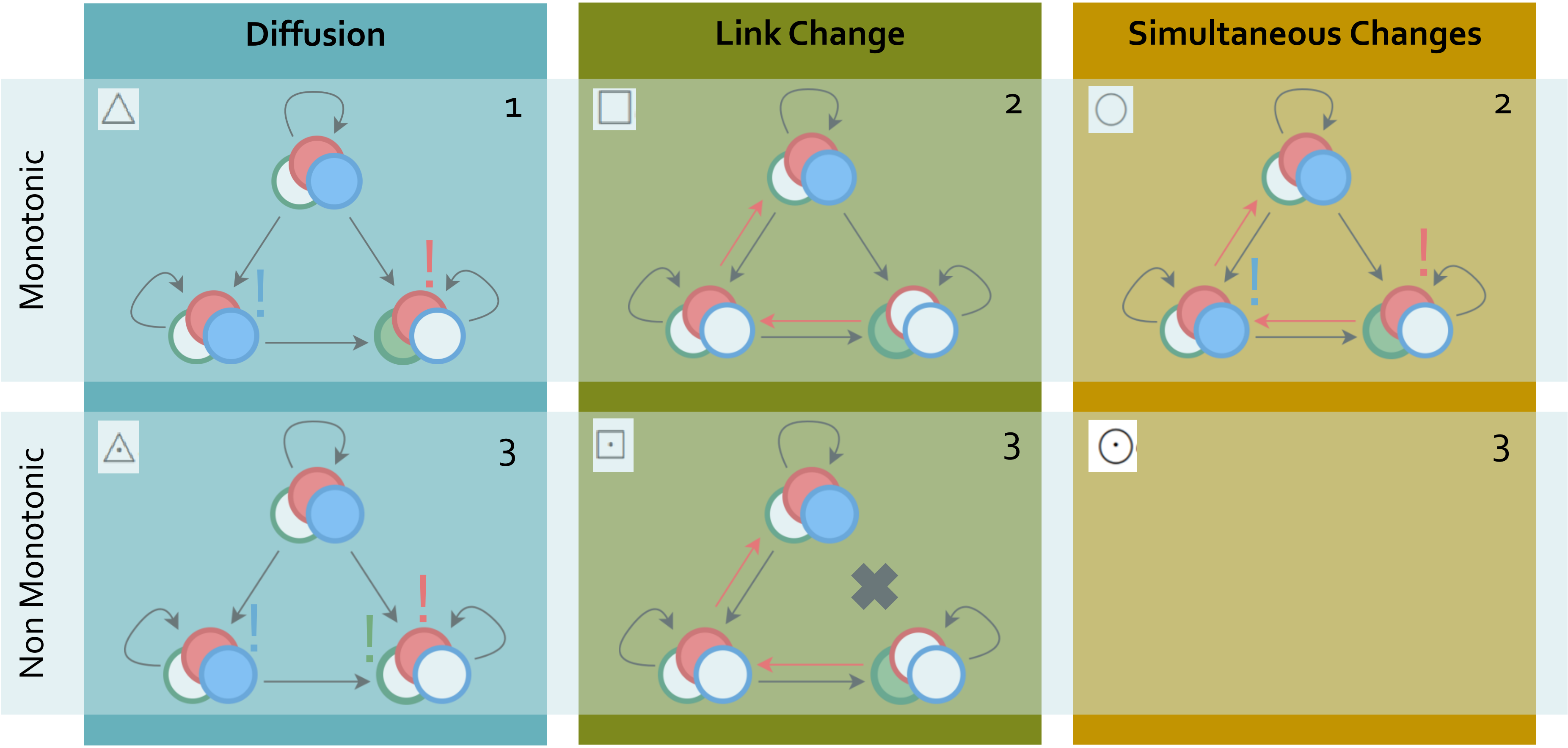

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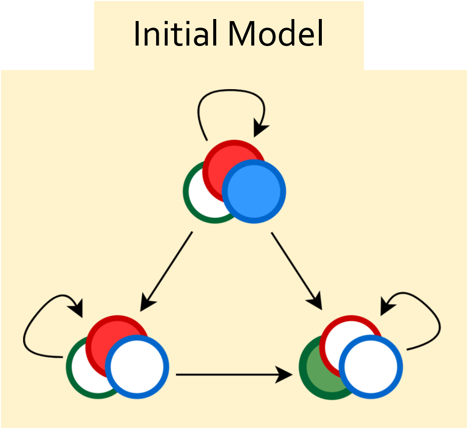

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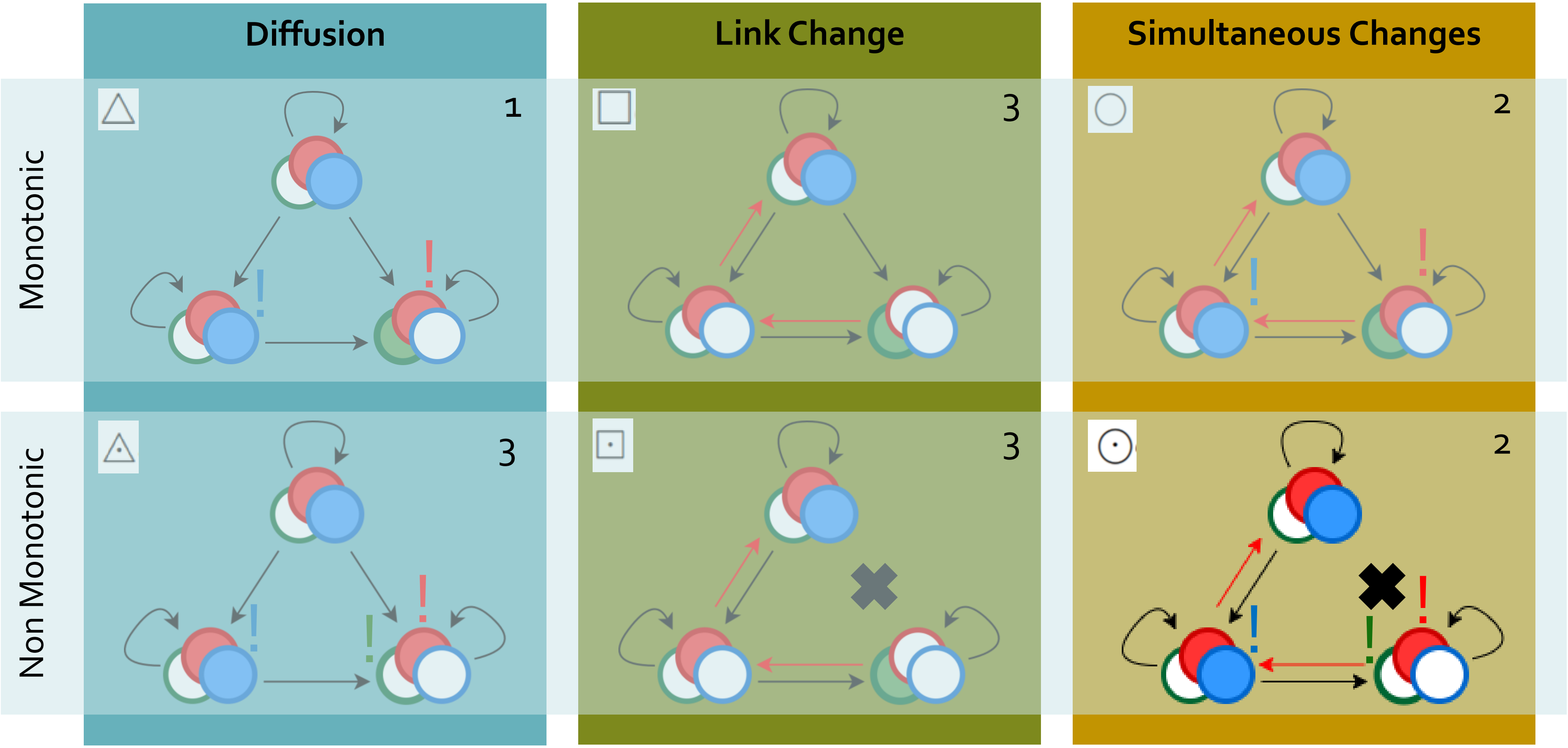

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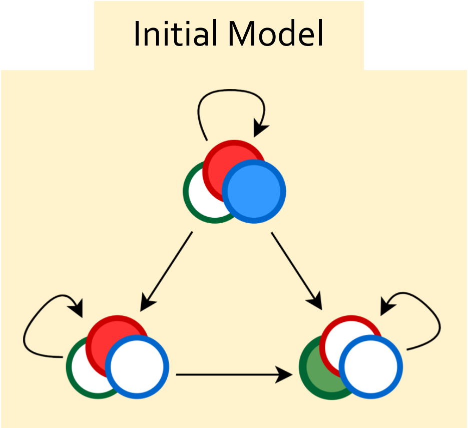

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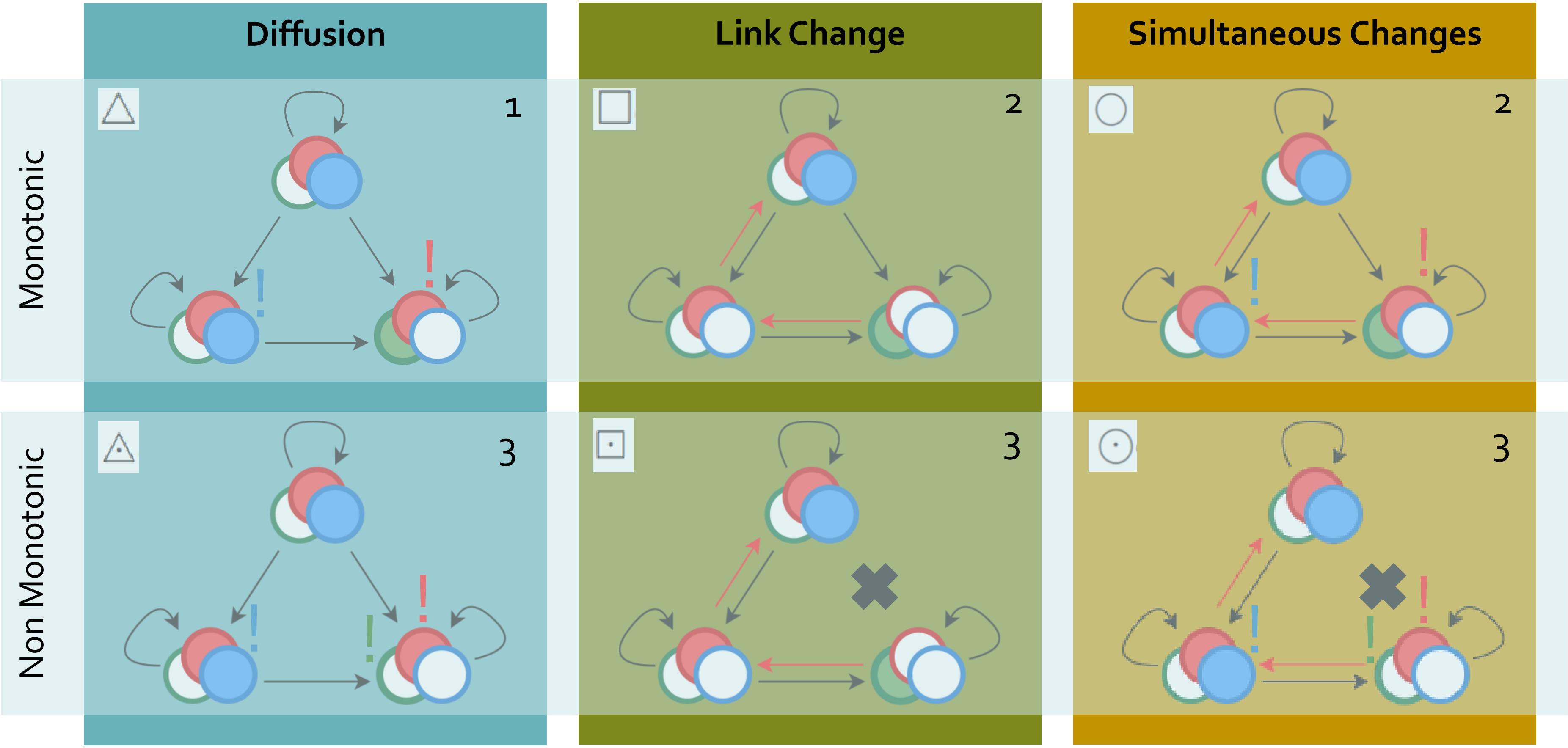

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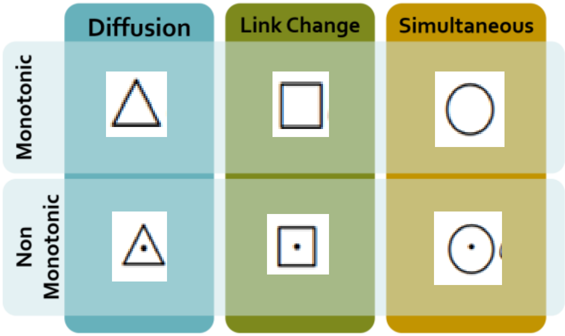
Syntax and semantics



$$\varphi := \; N_{ab} \mid f_a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \triangle\varphi \mid \dot{\triangle}\varphi \mid \Box\varphi \mid \boxdot\,\varphi \mid \bigcirc\,\varphi \mid \odot\varphi$$

$$
\begin{aligned}
&M \models f_a \textit{ if and only if } f \in V(a)\\
&M \models N_{ab} \textit{ if and only if } (a, b) \in N\\
&M \models \neg\varphi \textit{ if and only if } M \not\models \varphi\\
&M \models \varphi \wedge \psi \textit{ if and only if } M \models \varphi \textit{ and } M \models \psi\\
&M \models \mathsf{X}\varphi \textit{ if and only if } M_{\mathsf{X}} \models \varphi
\end{aligned}
$$

	Diffusion	Link Change	Simultaneous
Monotonic	$\triangle$	$\square$	$\bigcirc$
Non Monotonic	$\dot{\triangle}$	$\boxdot$	$\odot$

Syntax and semantics

$$\varphi := \ N_{ab} \mid f_a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \triangle\varphi \mid \dot{\triangle}\varphi \mid \square\varphi \mid \boxdot\,\varphi \mid \bigcirc\,\varphi \mid \odot\varphi$$

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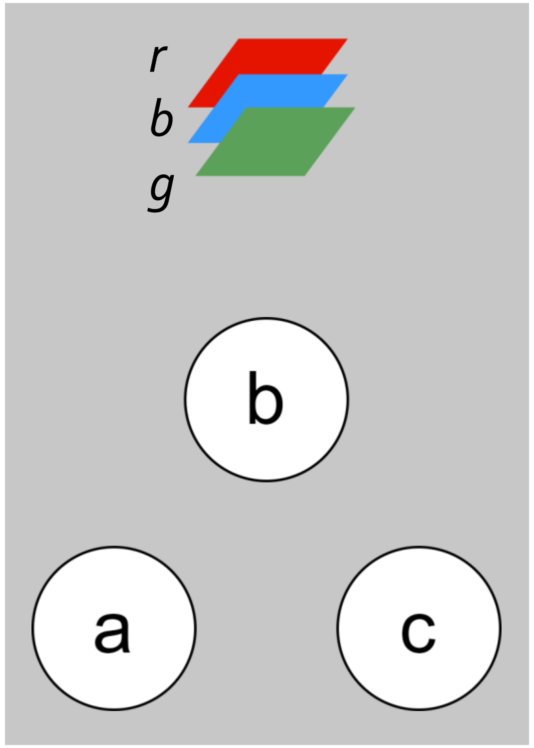
Dynamic operator

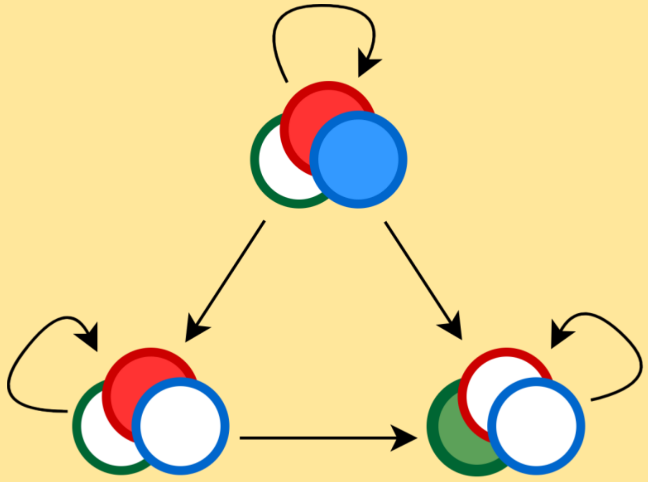
Model update corresponding to dynamic operator

Syntax and semantics

	Diffusion	Link Change	Simultaneous
Monotonic	$\triangle$	$\square$	$\bigcirc$
Non Monotonic	$\dot{\triangle}$	$\boxdot$	$\odot$

$$\varphi := \; N_{ab} \mid f_a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \triangle\varphi \mid \dot{\triangle}\varphi \mid \square\varphi \mid \boxdot\,\varphi \mid \bigcirc\,\varphi \mid \odot\varphi$$





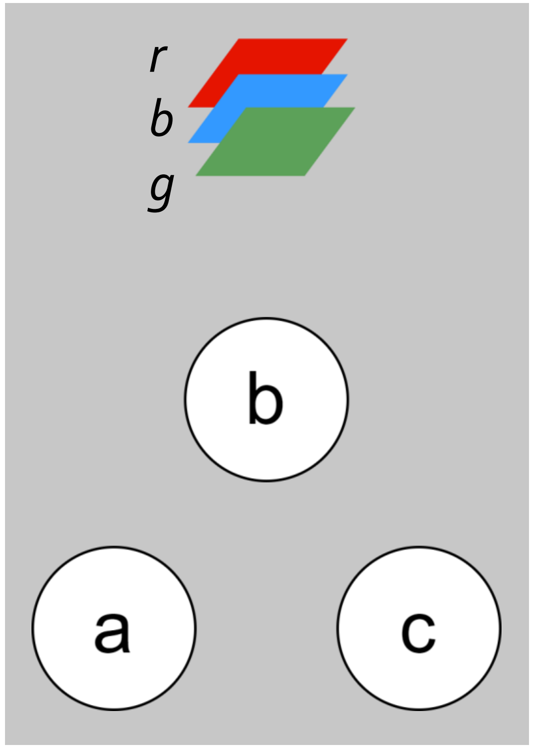
M

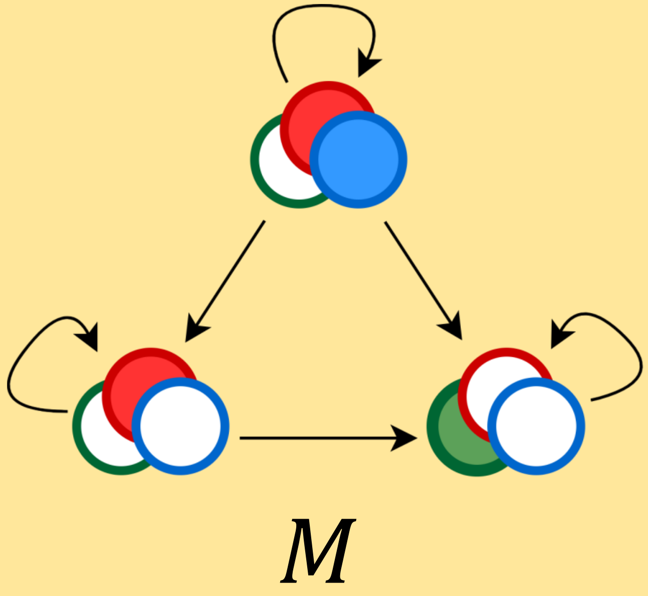
$$M \vDash \neg r_b \wedge N_{ba}$$

Syntax and semantics

	Diffusion	Link Change	Simultaneous
Monotonic	$\triangle$	$\Box$	$\bigcirc$
Non Monotonic	$\dot{\triangle}$	$\boxdot$	$\odot$

$$\varphi := \; N_{ab} \mid f_a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \triangle\varphi \mid \dot{\triangle}\varphi \mid \Box\varphi \mid \boxdot\,\varphi \mid \bigcirc\,\varphi \mid \odot\varphi$$

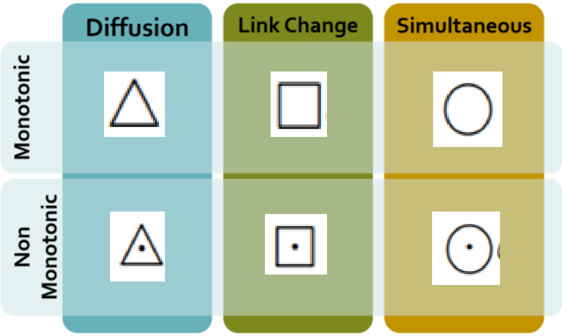




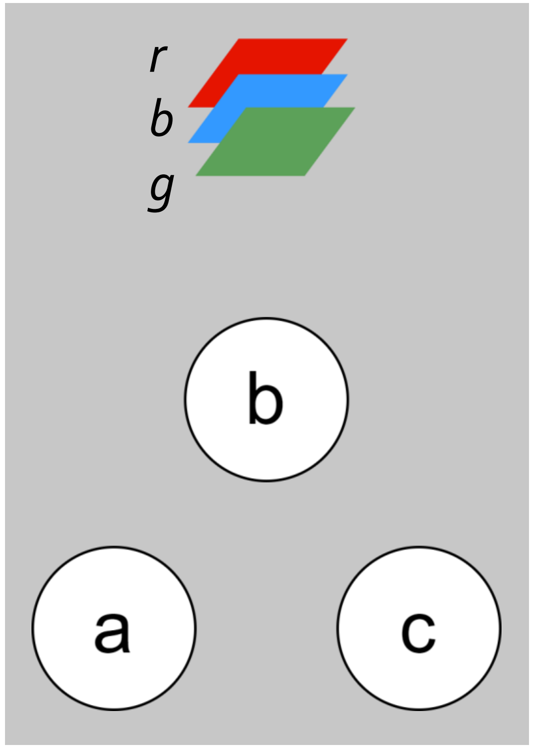
$$M \vDash \neg r_b \wedge N_{ba}$$

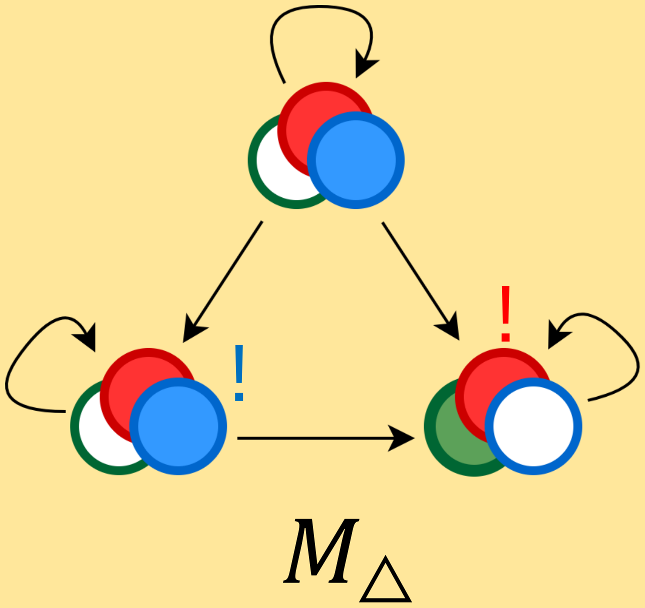
$$M \vDash \triangle\,(r_c \wedge N_{ba})$$

Syntax and semantics



$$\varphi := \; N_{ab} \mid f_a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \triangle\varphi \mid \dot{\triangle}\varphi \mid \Box\varphi \mid \boxdot\,\varphi \mid \bigcirc\,\varphi \mid \odot\varphi$$





$M \vDash \neg r_b \wedge N_{ba}$

$M_{\triangle} \vDash r_c \wedge N_{ba}$

Expressing social pressures

Conformity (adoption) pressure

$$f^{\tau}_{N(a)} := \bigvee_{\{G \subseteq N \subseteq A,\ N \neq \emptyset\ :\ \frac{|G|}{|N|} \geq \tau\}} \left(\bigwedge_{b \in N} N_{ba} \wedge \bigwedge_{b \notin N} \neg N_{ba} \wedge \bigwedge_{b \in G} f_b \right)$$

Similarity pressure

$$sim^{\omega}_{ab} := \dot{\bigvee}_{\{E \subseteq F : \frac{|E|}{|F|} \geq \omega\}} \bigwedge_{f \in E} (f_a \leftrightarrow f_b)$$

Axiom System

	Diffusion	Link Change	Simultaneous
Monotonic	$\triangle$	$\square$	$\bigcirc$
Non Monotonic	$\dot{\triangle}$	$\boxdot$	$\odot$

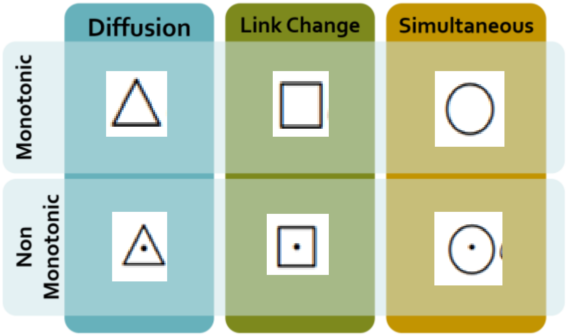
$\square N_{ab} \leftrightarrow N_{ab} \vee sim^{\omega}_{ab}$	$1_{\square}$	$\boxdot N_{ab} \leftrightarrow sim^{\omega}_{ab}$	$1_{\boxdot}$
$\square f_a \leftrightarrow f_a$	$2_{\square}$	$\boxdot f_a \leftrightarrow f_a$	$2_{\boxdot}$
$\square(\varphi \wedge \psi) \leftrightarrow \square\varphi \wedge \square\psi$	$3_{\square}$	$\boxdot(\varphi \wedge \psi) \leftrightarrow \boxdot\varphi \wedge \boxdot\psi$	$3_{\boxdot}$
$\square\neg\varphi \leftrightarrow \neg\square\varphi$	$4_{\square}$	$\boxdot\neg\varphi \leftrightarrow \neg\boxdot\varphi$	$4_{\boxdot}$
$\triangle N_{ab} \leftrightarrow N_{ab}$	$1_{\triangle}$	$\dot{\triangle} N_{ab} \leftrightarrow N_{ab}$	$1_{\dot{\triangle}}$
$\triangle f_a \leftrightarrow f_a \vee f^{\tau}_{N(a)}$	$2_{\triangle}$	$\dot{\triangle} f_a \leftrightarrow (f_a \wedge \neg \bigvee_{b \in A} N_{ba}) \vee f^{\tau}_{N(a)}$	$2_{\dot{\triangle}}$
$\triangle(\varphi \wedge \psi) \leftrightarrow \triangle\varphi \wedge \triangle\psi$	$3_{\triangle}$	$\dot{\triangle}(\varphi \wedge \psi) \leftrightarrow \dot{\triangle}\varphi \wedge \dot{\triangle}\psi$	$3_{\dot{\triangle}}$
$\triangle\neg\varphi \leftrightarrow \neg\triangle\varphi$	$4_{\triangle}$	$\dot{\triangle}\neg\varphi \leftrightarrow \neg\dot{\triangle}\varphi$	$4_{\dot{\triangle}}$
$\bigcirc N_{ab} \leftrightarrow N_{ab} \vee sim^{\omega}_{ab}$	$1_{\bigcirc}$	$\odot N_{ab} \leftrightarrow sim^{\omega}_{ab}$	$1_{\odot}$
$\bigcirc f_a \leftrightarrow f_a \vee f^{\tau}_{N(a)}$	$2_{\bigcirc}$	$\odot f_a \leftrightarrow (f_a \wedge \neg \bigvee_{b \in A} N_{ba}) \vee f^{\tau}_{N(a)}$	$2_{\odot}$
$\bigcirc(\varphi \wedge \psi) \leftrightarrow \bigcirc\varphi \wedge \bigcirc\psi$	$3_{\bigcirc}$	$\odot(\varphi \wedge \psi) \leftrightarrow \odot\varphi \wedge \odot\psi$	$3_{\odot}$
$\bigcirc\neg\varphi \leftrightarrow \neg\bigcirc\varphi$	$4_{\bigcirc}$	$\odot\neg\varphi \leftrightarrow \neg\odot\varphi$	$4_{\odot}$
If $\varphi_1 \leftrightarrow \varphi_2$, infer that $\varphi \leftrightarrow \varphi[\varphi_1/\varphi_2]$, where $\varphi[\varphi_1/\varphi_2]$ is a formula obtained by replacing one or more occurrences of φ_1 with φ_2	Subs.	If $\varphi_1 \rightarrow \varphi_2$ and φ_1, infer that φ_2	MP

Comparing Social Network Dynamics

2. Comparing dynamics via replaceability

2.1 General irreplaceability result

Replaceability

$D = \{$  $\}$

Sequences of operators

Fix a subset $O \subseteq D$:

- S_O contains **all non-empty finite sequences** induced by the operators in O.

Replaceability

Equivalence between sequences

$$s_1, s_2 \text{ are equivalent iff } Ms_1 = Ms_2$$

Replaceability

Equivalence between sequences

$$s_1, s_2 \text{ are equivalent iff } Ms_1 = Ms_2$$

Replaceability of sets of sequences

Fix two subset $O_1, O_2 \subseteq D$:

S_{O_1} is replaceable by S_{O_2} on M iff for each $s_1 \in S_{O_1}$ there is a sequence $s_2 \in S_{O_2}$ equivalent to it on M.

Irreplaceability

Irreplaceability of a sequence set

S_{O1} is **irreplaceable** iff there is no $O_2 \subseteq D$ such that:

- $O_1 \nsubseteq O_2$
- S_{O_1} is replaceable by S_{O_2} over the class of all models.

Irreplaceability

General Irreplaceability

Fix non-empty $O \subseteq D$.Then, $\boldsymbol{S_O}$ **is irreplaceable.**

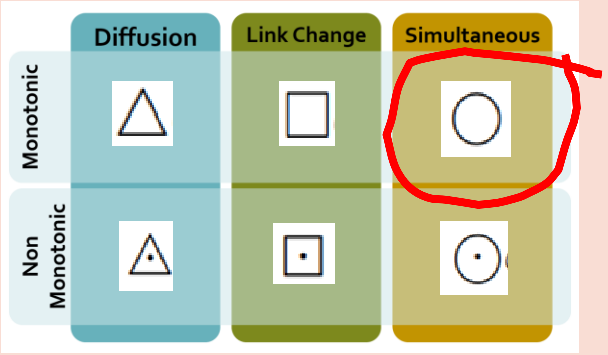
For any set of operators, we might pick:

- there exists a sequence
- and a model to which I can apply it
- whose effects cannot be emulated using any sequences of other operators.

Irreplaceability: The proof

Fix an operator

General Irreplaceability



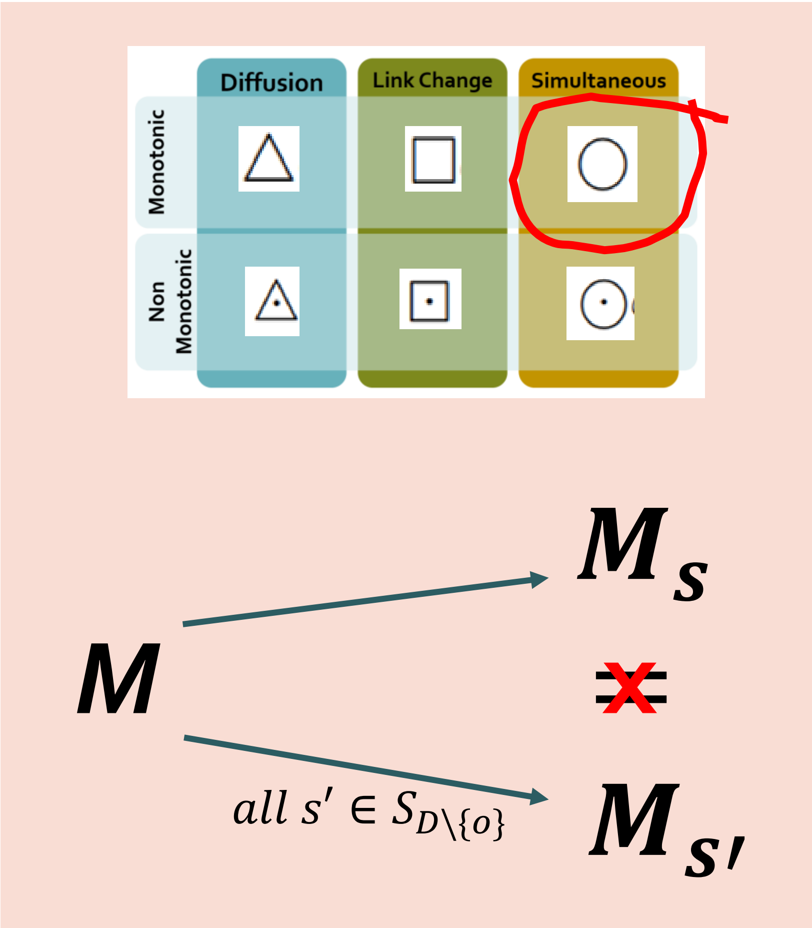
Irreplaceability: The proof

Fix an operator

Find a sequence s in $S_{\{o\}}$ such that:

s has no equivalent in $S_{D\backslash\{o\}}$ on some model M (M is a countermodel!)

General Irreplaceability



Irreplaceability: The proof

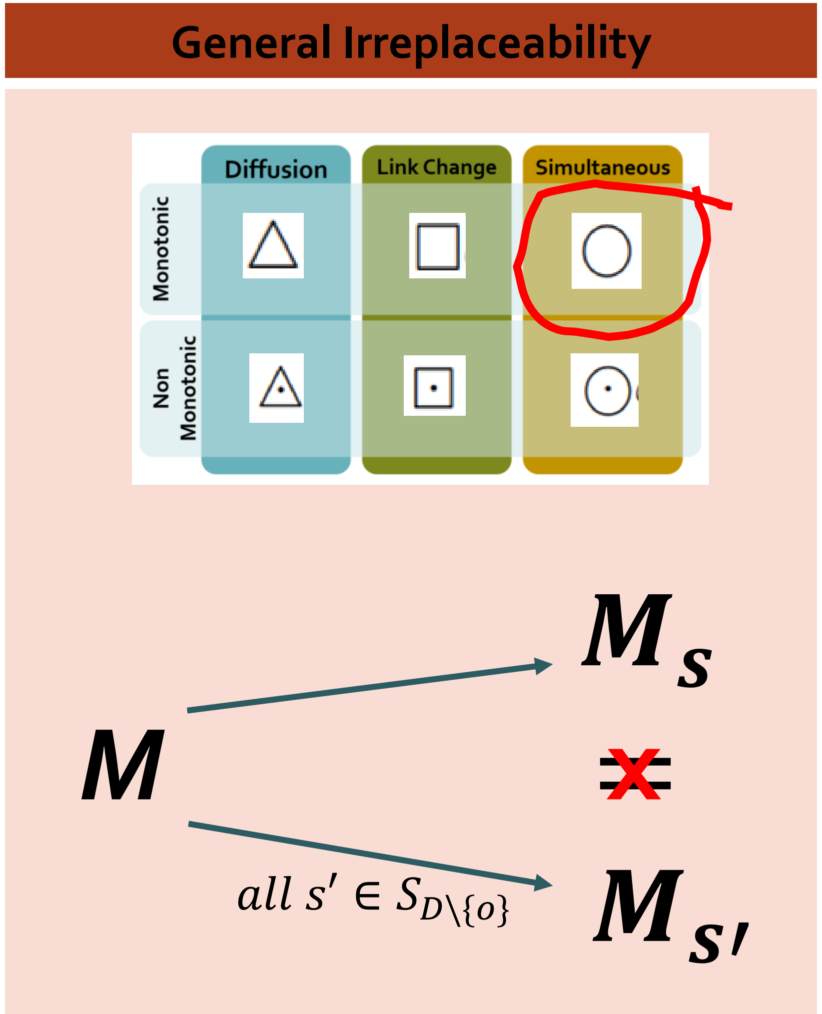
Fix an operator

Find a sequence s in $S_{\{o\}}$ such that:

s has no equivalent in $S_{D\setminus\{o\}}$ on some model M (M is a countermodel!)

By subsets of $S_{D\setminus\{o\}}$:

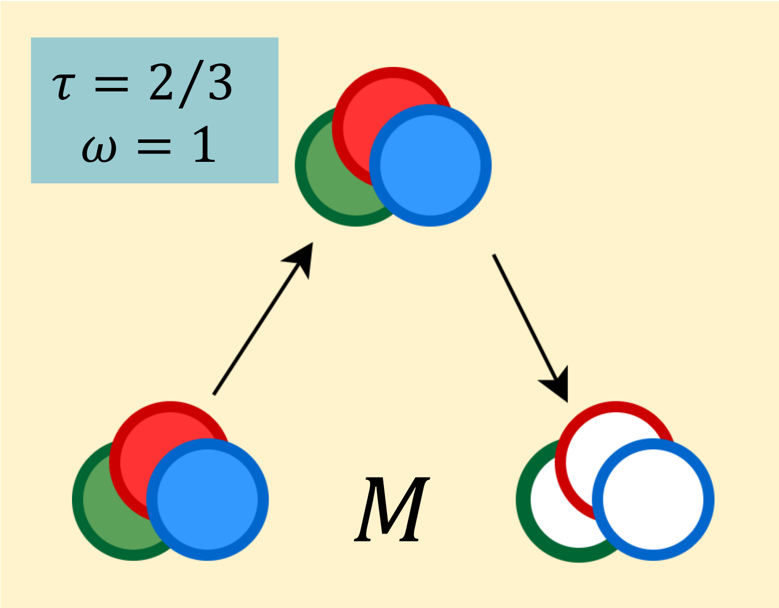
Conclude irreplaceability $S_{\{o\}}$



(**Part of**) The case of ○

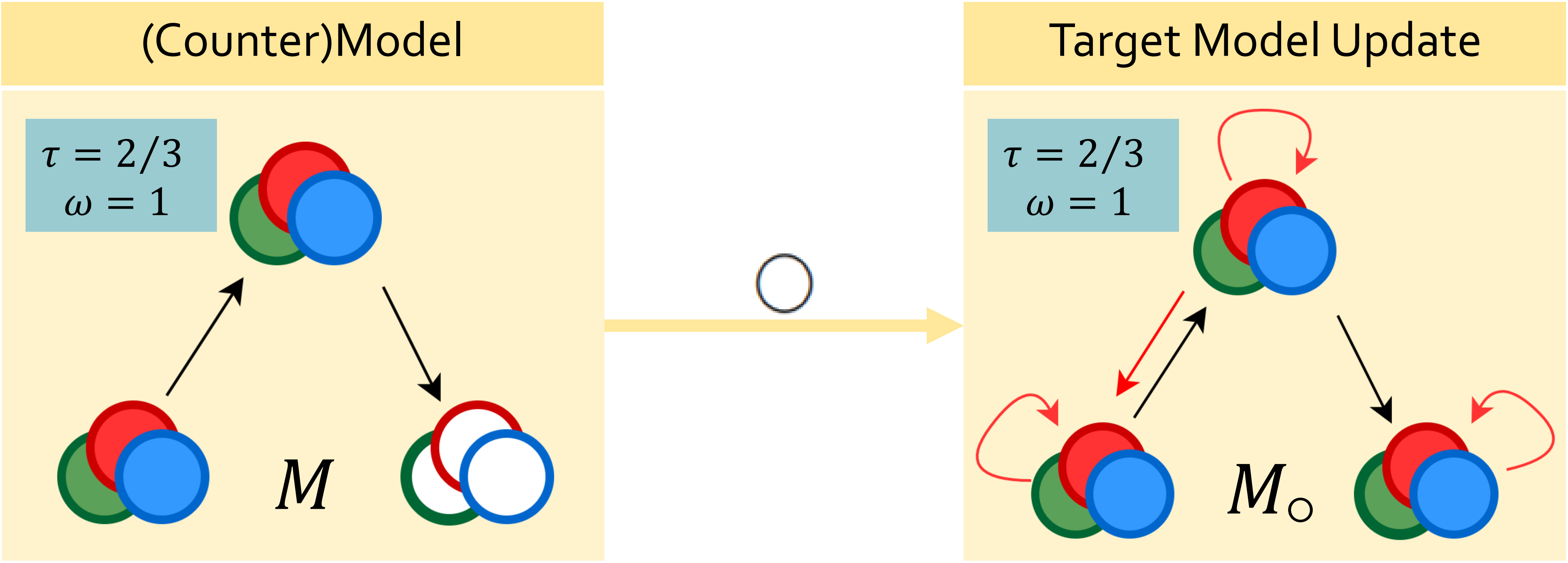
(Irreplaceable) Sequence ○

(Counter)Model



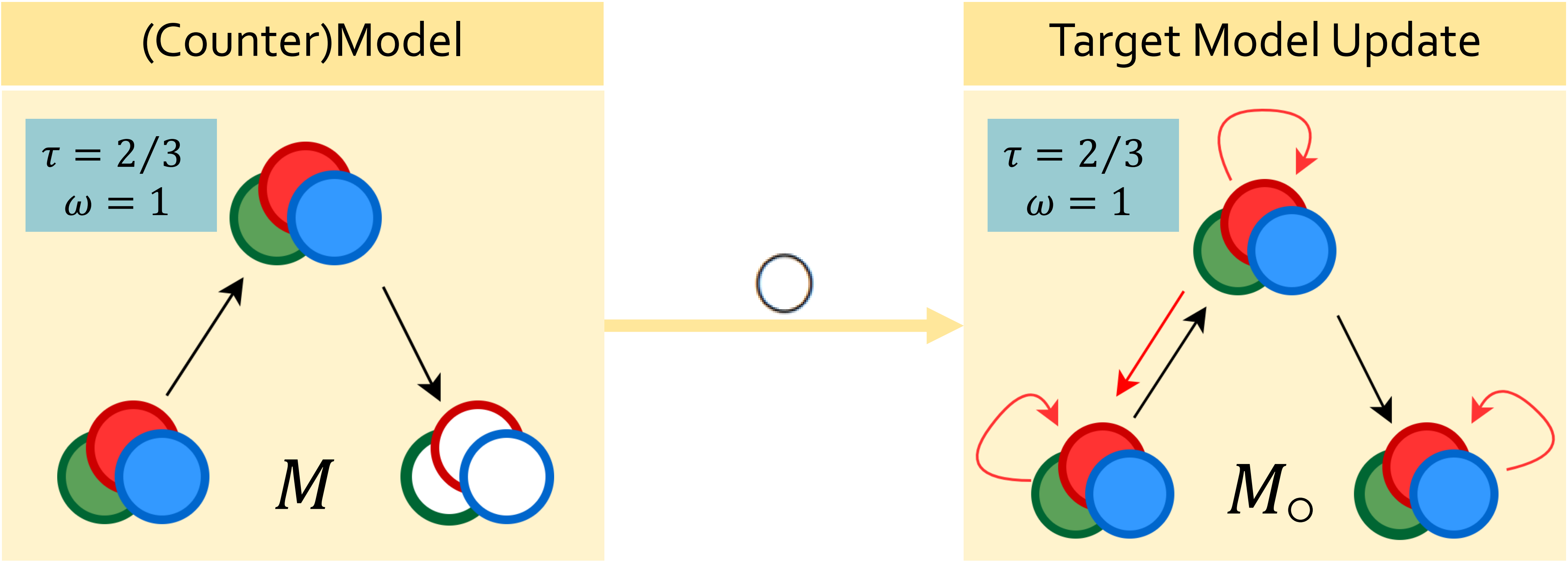
(**Part of**) The case of ○

(Irreplaceable) Sequence ○



(**Part of**) The case of $\bigcirc$

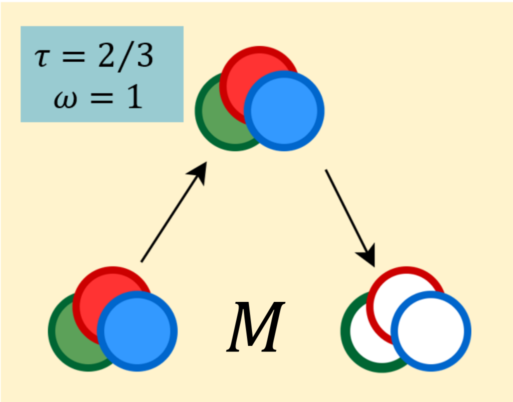
(Irreplaceable) Sequence $\circ$



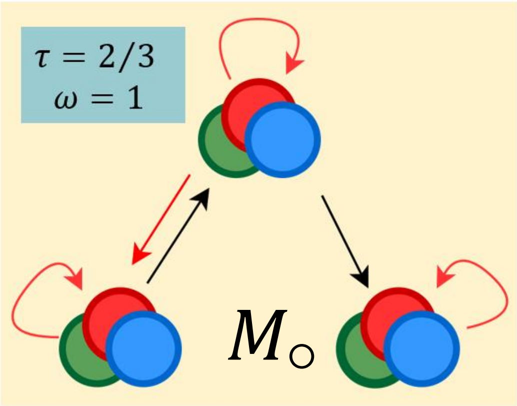
Can we obtain $M_\circ$ from M using other operators in a sequence?

Can we obtain $M_\circ$ from M using other operators in a sequence?

(Counter)Model

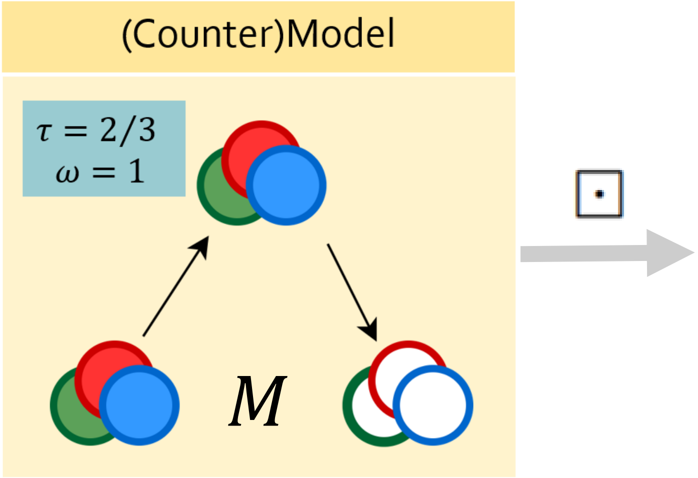


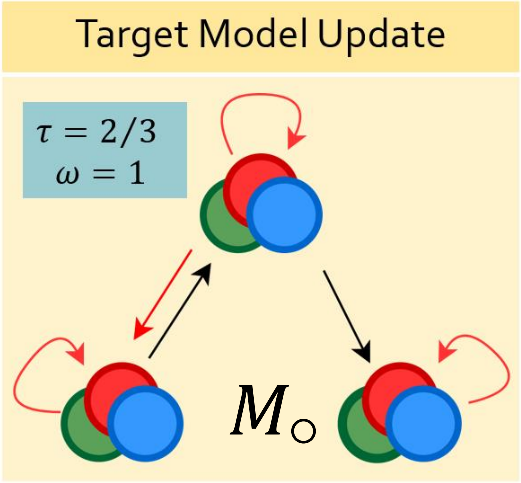
Target Model Update



Can we obtain $M_{\circ}$ from M using other operators in a sequence?

Consider a sequence that starts with $\boxdot$.





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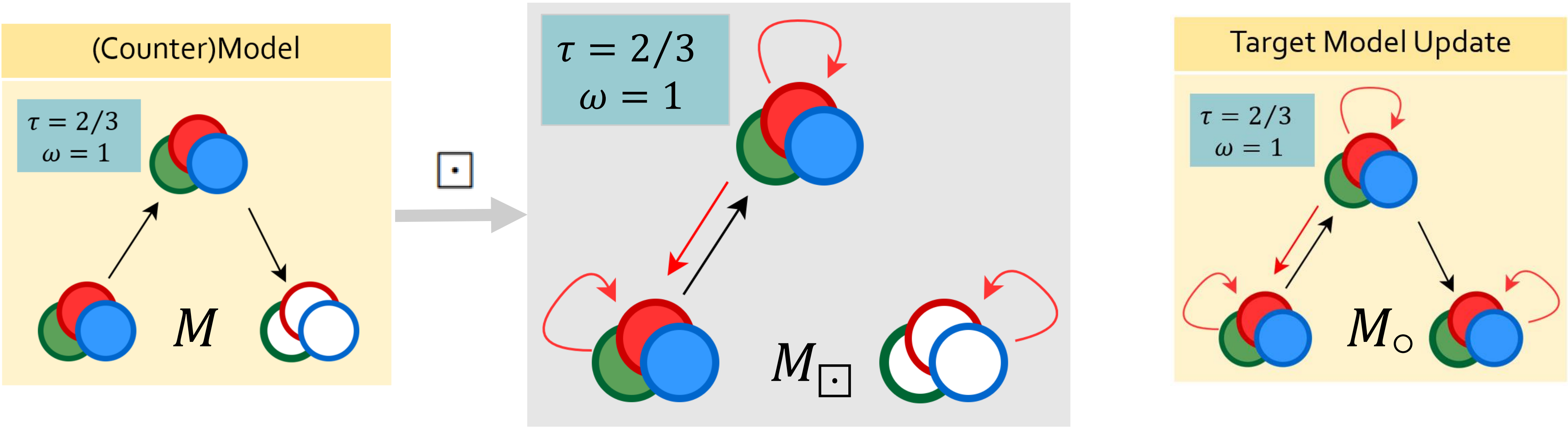

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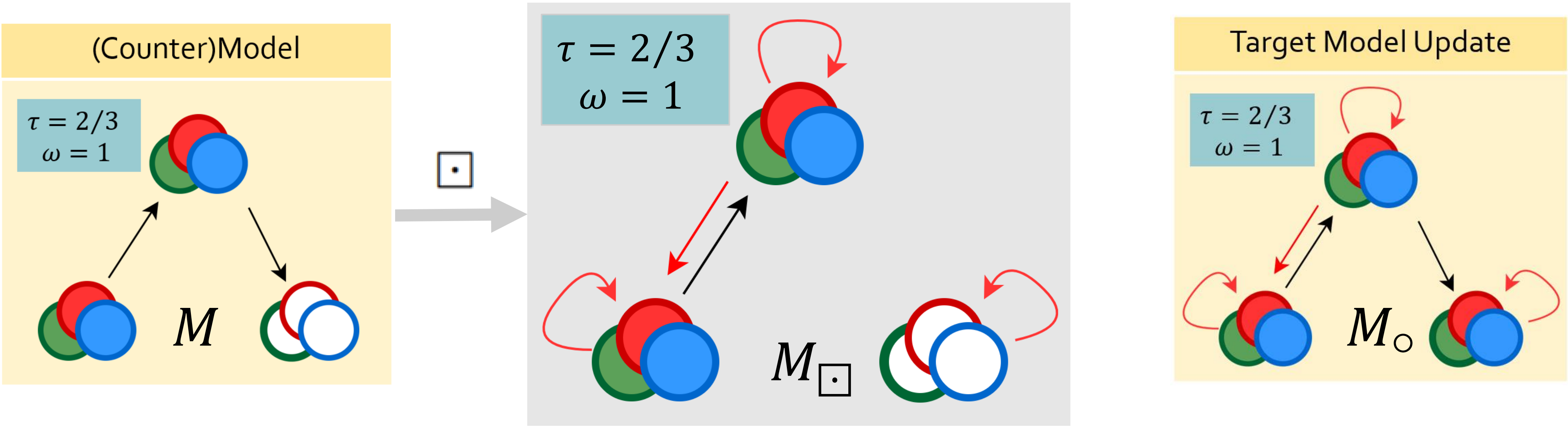
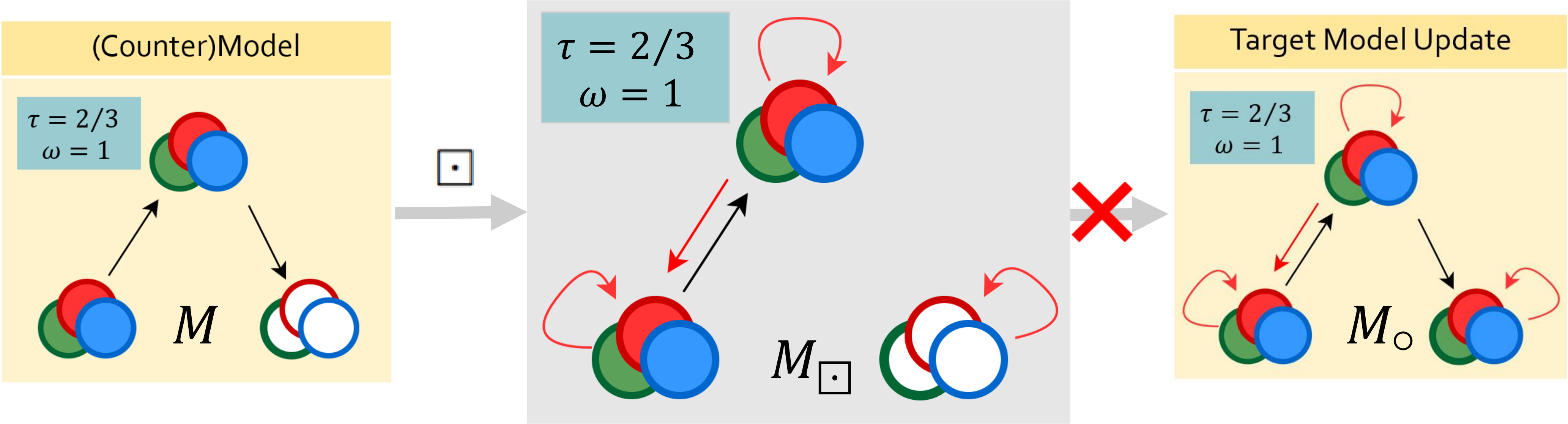


This model will never change!

Can we obtain $M_{\circ}$ from M using other operators in a sequence?

Consider a sequence that starts with $\boxdot$.



This model will never change!

Can we obtain $M_\circ$ from M using other operators in a sequence?

No sequence starting with $\boxdot$ can reach $M_\circ$ from M.

Can we obtain $M_\circ$ from M using other operators in a sequence?

No sequence starting with $\boxdot$ can reach $M_\circ$ from M.

Consider a sequence that starts with $\odot$.

Can we obtain $M_{\circ}$ from M using other operators in a sequence?

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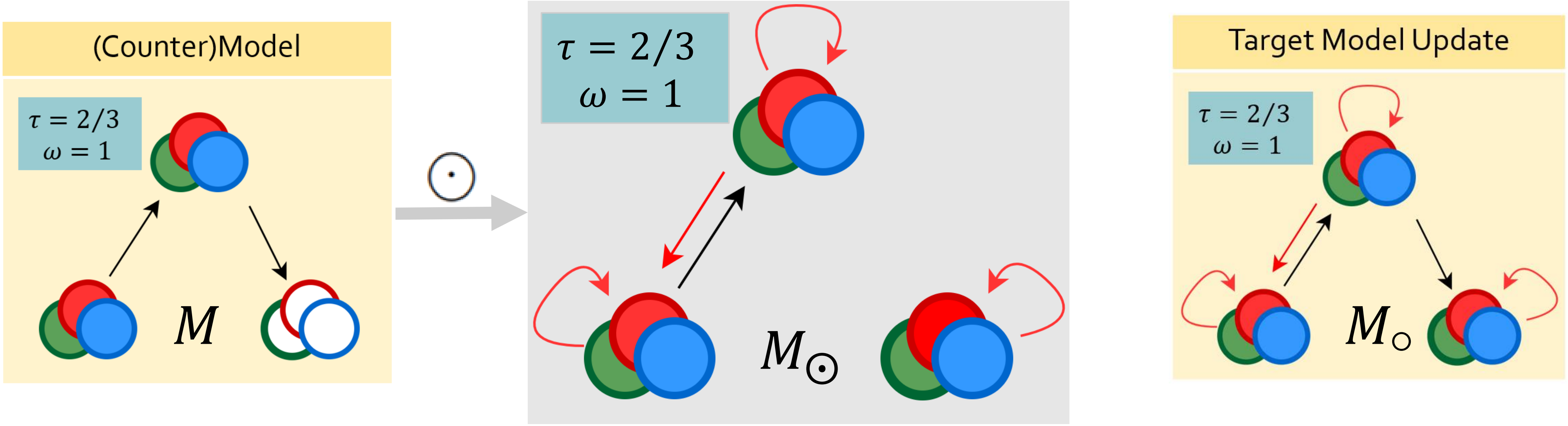

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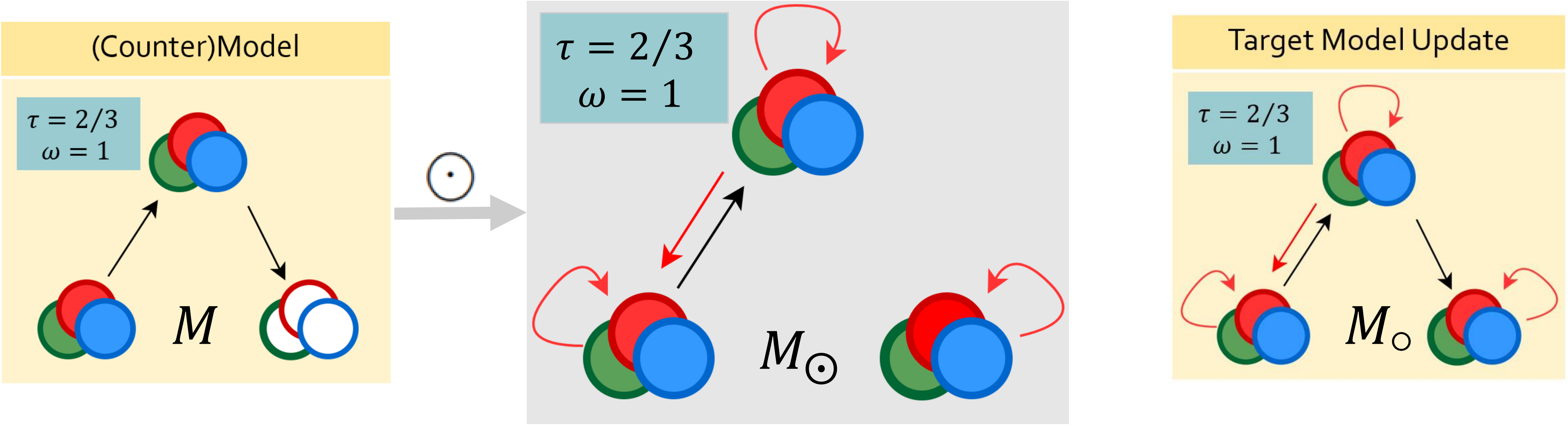


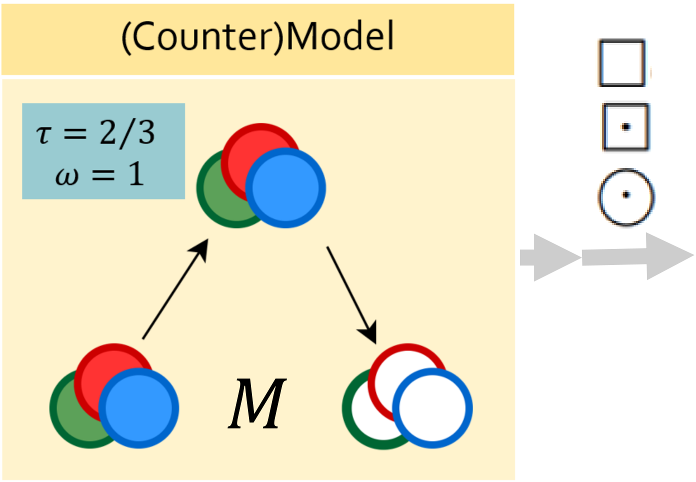
Opinions are stable.

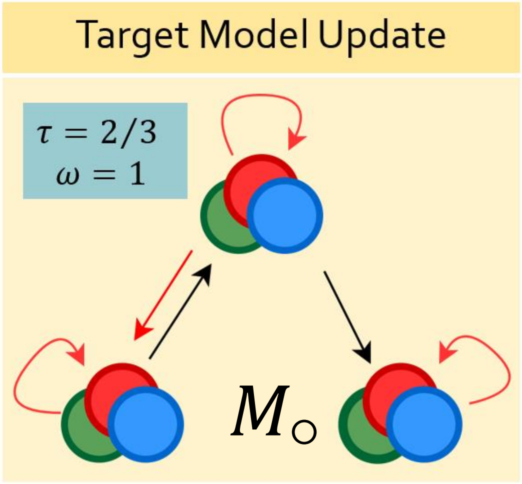
Try use network change.

Can we obtain $M_\circ$ from M using other operators in a sequence?

No sequence starting with $\boxdot$ can reach $M_\circ$ from M.

Consider a sequence that starts with $\odot$.





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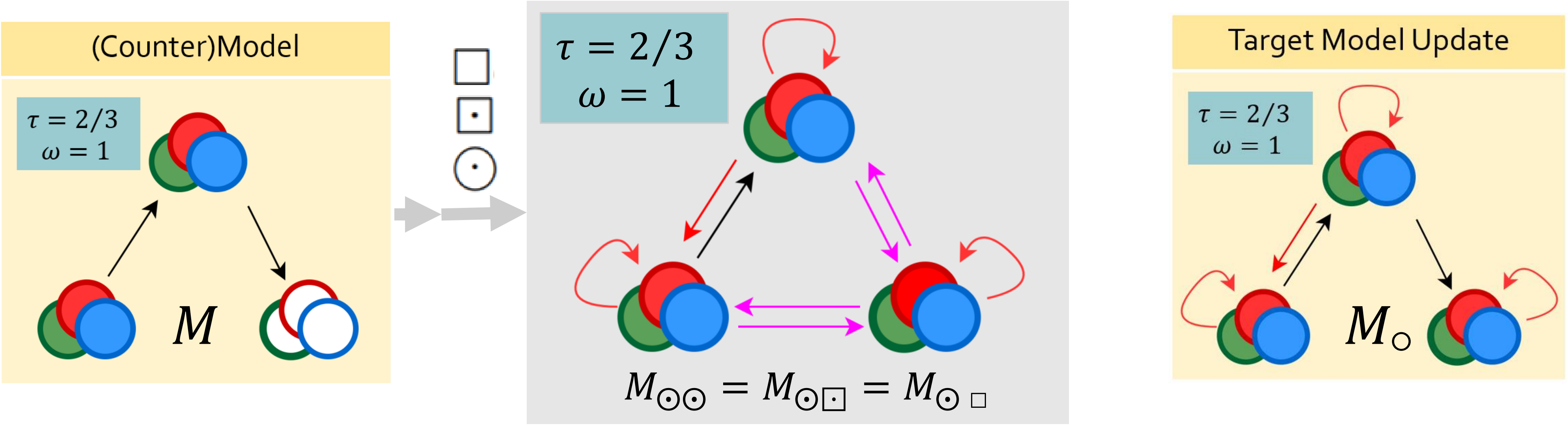

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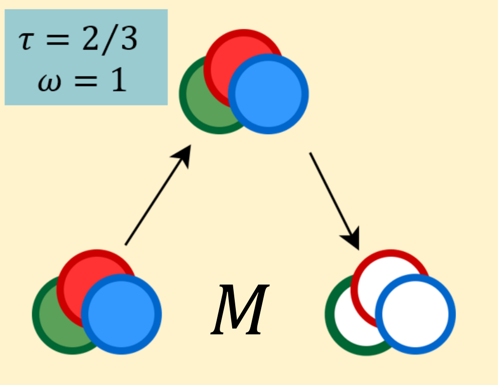
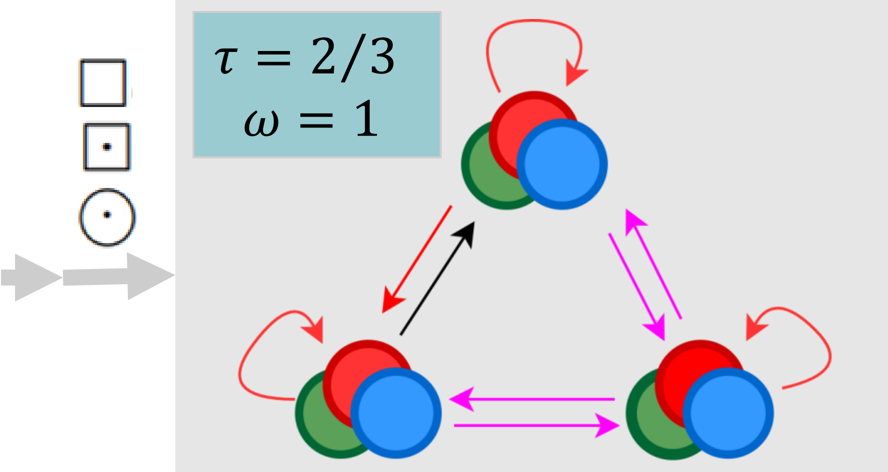
Consider a sequence that starts with $\odot$.

(Counter)Model

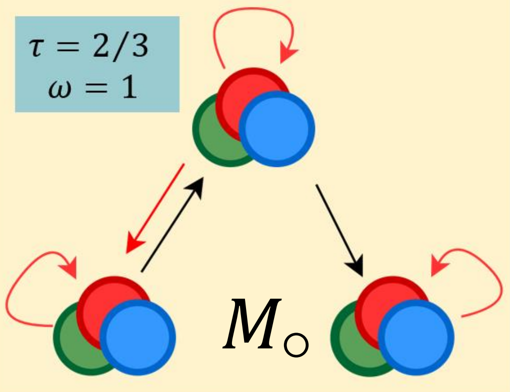




$$M_{\odot\odot} = M_{\odot\boxdot} = M_{\odot\,\square}$$

This model will never change!

Target Model Update



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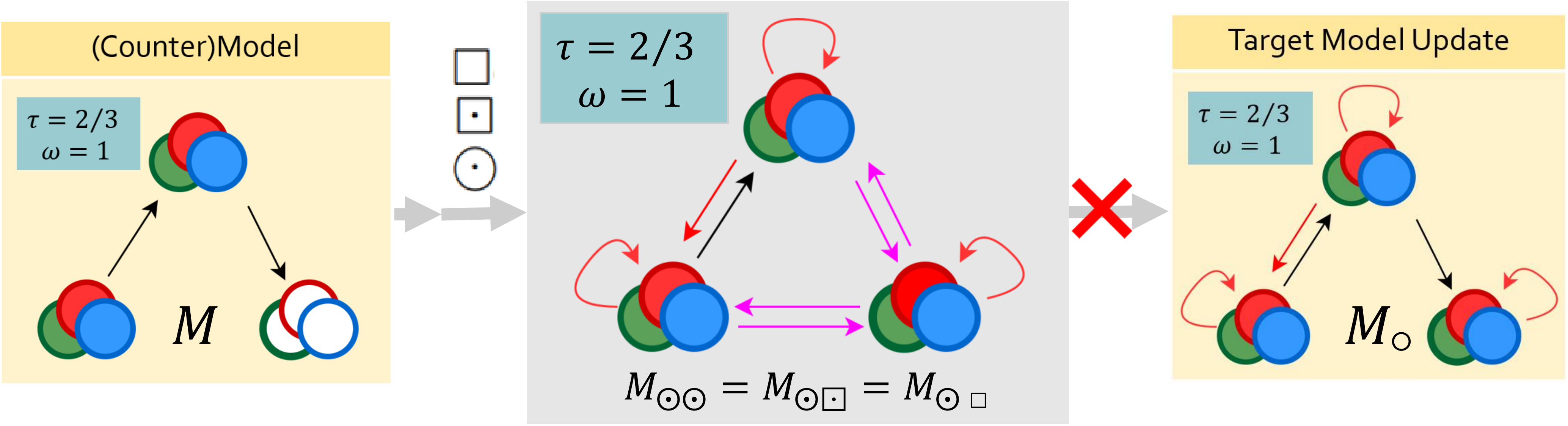

This model will never change!

Can we obtain $M_{\circ}$ from M using other operators in a sequence?

No sequence starting with $\boxdot$ can reach $M_{\circ}$ from M.

No sequence starting with $\odot$ can reach $M_{\circ}$ from M.

Can we obtain $M_{\bigcirc}$ from M using other operators in a sequence?

No sequence starting with $\boxdot$ can reach $M_{\bigcirc}$ from M.

No sequence starting with $\odot$ can reach $M_{\bigcirc}$ from M.

...

No sequence in $S_{D\setminus\{o\}}$ can replace $\bigcirc$ on M.

Can we obtain $M_\circ$ from M using other operators in a sequence?

No sequence starting with $\boxdot$ can reach $M_\circ$ from M.

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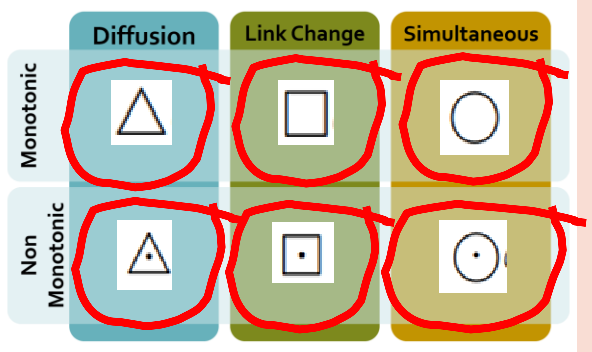
…

No sequence in $S_{D\setminus\{o\}}$ can replace $\bigcirc$ on M.

Do this for all operators in D.

General Irreplaceability



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...

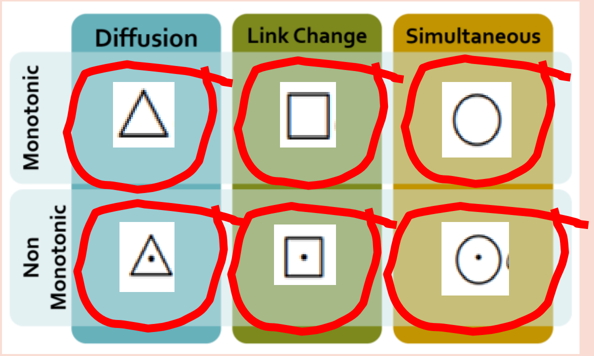
No sequence in $S_{D\setminus\{o\}}$ can replace $\bigcirc$ on M.

Do this for all operators in D.

By superset of each $S_{\{o\}}$:

Conclude irreplaceability any S_O.

General Irreplaceability



Can we obtain $M_\circ$ from M using other operators in a sequence?

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…

No sequence in $S_{D\setminus\{o\}}$ can replace $\bigcirc$ on M.

Do this for all operators in D.

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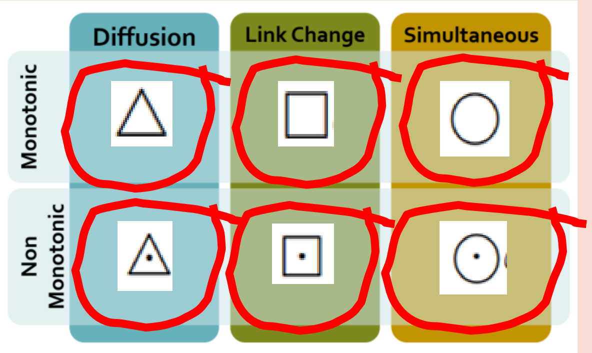
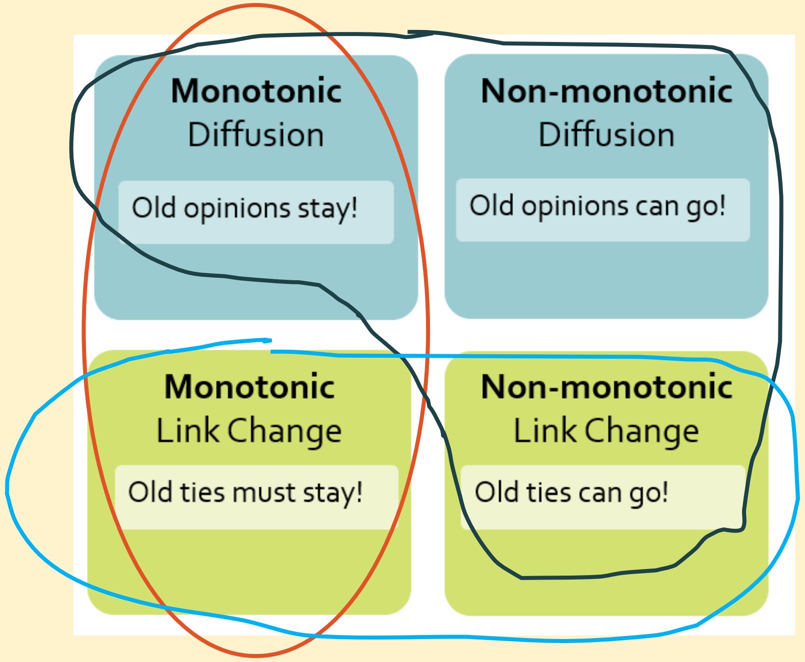
Conclude irreplaceability any S_O.

General Irreplaceability



Comparing Social Network Dynamics

Model Analysis



Q	Do different sets capture different dynamics?
Q	Can we reduce sets of operations to others?

Comparing Social Network Dynamics

Q

Can different sets capture different dynamics?

Q

Can we reduce sets of operations to others?

Comparing Social Network Dynamics

Q

Can different sets capture different dynamics?

In general, yes.

Q

Can we reduce sets of operations to others?

Not always.

General Irreplaceability

Fix non-empty $O \subseteq D$.Then, $\boldsymbol{S_O}$ **is irreplaceable.**

Comparing Social Network Dynamics

Q

Can different sets capture different dynamics?

In general, yes.

Q

Can we reduce sets of operations to others?

Not always.

Q

Can you sometimes replace a sequence set? When?

General Irreplaceability

Fix non-empty $O \subseteq D$.Then, $\boldsymbol{S_O}$ **is irreplaceable.**

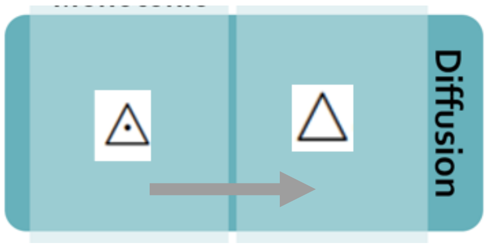
Comparing Social Network Dynamics

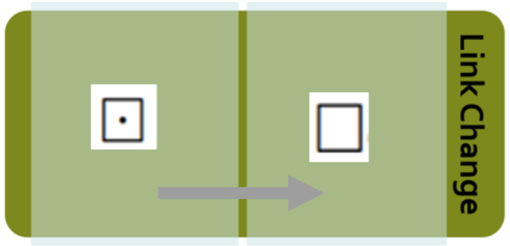
2. Comparing dynamics via replaceability

2.2 Replaceability on special classes of models

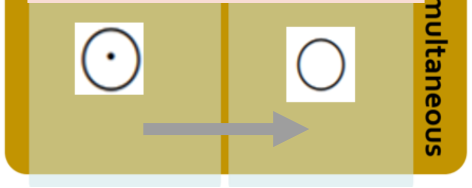
Replaceability on special model classes

1. Replaceability of non-monotonic changes by the corresponding monotonic changes.



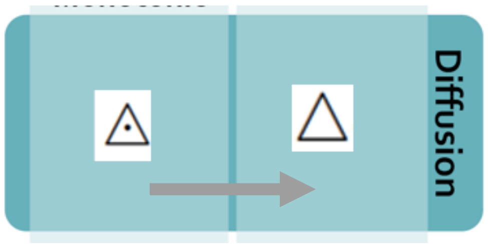


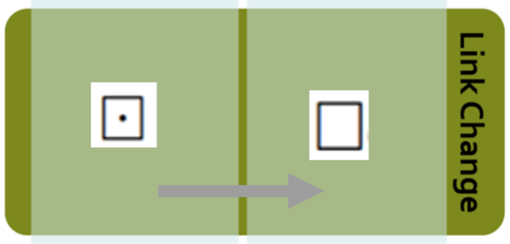
Issues with oscillations!

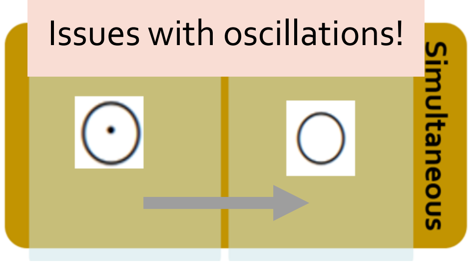


Replaceability on special model classes

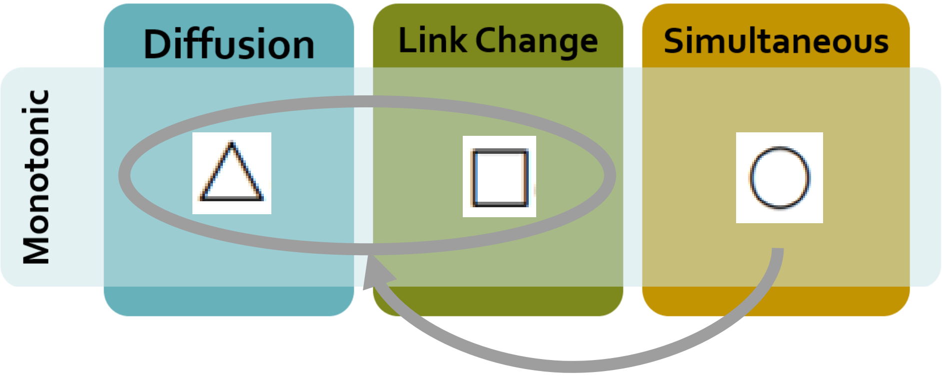
1. Replaceability of non-monotonic changes by the corresponding monotonic changes.

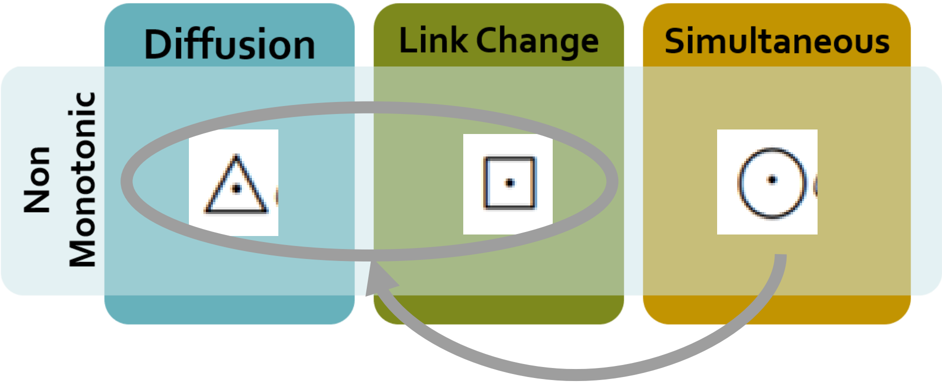




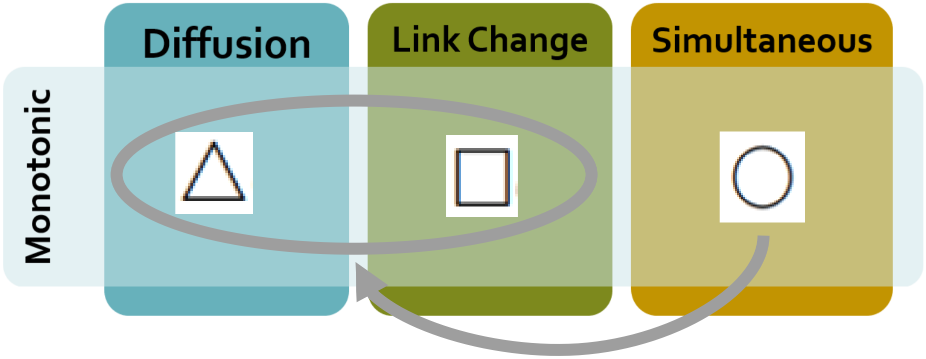


2. Replaceability of simultaneous changes by the corresponding non-simultaneous changes.





Replaceability on special model classes



Replacing $S_{\{\bigcirc\}}$ (a sufficient condition)

1. **Characterisation** of the class of models where the sequence $\bigcirc$ is replaceable.

2. **Sufficient condition** for the replaceability of $S_{\{\bigcirc\}}$.

Replacing $S_{\{○\}}$

1. Characterisation of the class of models where the sequence ○ is replaceable.

Sequences that are equivalent to ○ on special classes of models:

Replacing $S_{\{○\}}$

1. Characterisation of the class of models where the sequence ○ is replaceable.

Sequences that are equivalent to ○ on special classes of models:

If **the network is stable**, simultaneous = diffusion. (○ ≡△)

Replacing $S_{\{\bigcirc\}}$

1. Characterisation of the class of models where the sequence ◯ is replaceable.

Sequences that are equivalent to ◯ on special classes of models:

If **the network is stable**, simultaneous = diffusion. (◯ ≡△)

$$\bigcirc \text{ is equivalent to } \triangle \text{ iff } M \models \bigwedge_{a,b\in A}(N_{ab} \vee \neg sim^{\omega}_{ab})$$

Replacing $S_{\{\bigcirc\}}$

1. Characterisation of the class of models where the sequence $\bigcirc$ is replaceable.

Sequences that are equivalent to $\bigcirc$ on special classes of models:

$\triangle$

Replacing $S_{\{○\}}$

1. Characterisation of the class of models where the sequence ○ is replaceable.

Sequences that are equivalent to ○ on special classes of models:

△

If **diffusion is stable**, simultaneous = network change. (○ ≡□)

Replacing $S_{\{○\}}$

1. Characterisation of the class of models where the sequence ○ is replaceable.

Sequences that are equivalent to ○ on special classes of models:

$\triangle$

If **diffusion is stable**, simultaneous = network change. (○ ≡□)

$$\bigcirc \text{ is equivalent to } \square \text{ iff } M \models \bigwedge_{a \in A} \bigwedge_{f \in F} (f_a \vee \neg f^{\tau}_{N(a)})$$

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Sequences that are equivalent to $\bigcirc$ on special classes of models:

$\triangle$ $\square$

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1. Characterisation of the class of models where the sequence $\bigcirc$ is replaceable.

Sequences that are equivalent to $\bigcirc$ on special classes of models:

$\triangle$ $\square$

If **a diffusion step does not change who is similar to whom**:

simultaneous = diffusion ↷ network change ($\bigcirc \equiv \triangle\square$).

$$\bigcirc \text{ is equivalent to } \triangle\square \text{ iff } M \models \bigwedge_{a,b\in A}(\neg N_{ab} \to (sim^{\omega}_{ab} \leftrightarrow \triangle sim^{\omega}_{ab}))$$

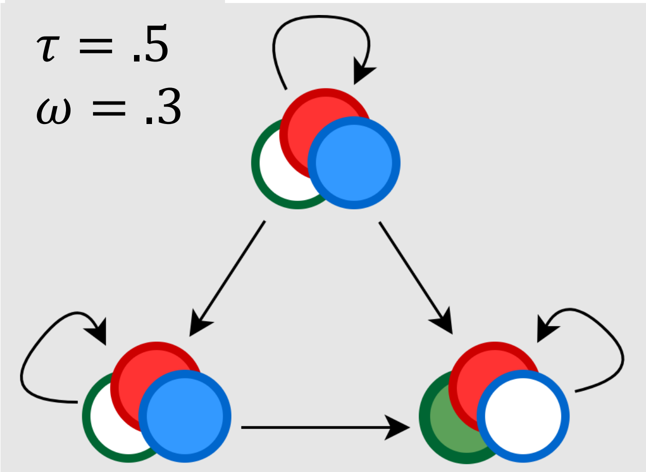
Replacing $S_{\{O\}}$

1. Characterisation of the class of models where the sequence ◯ is replaceable.

Sequences that are equivalent to ◯ on special classes of models:

△ □

If **a diffusion step does not change who is similar to whom**, simultaneous = diffusion ↷ network.



Replacing $S_{\{\bigcirc\}}$

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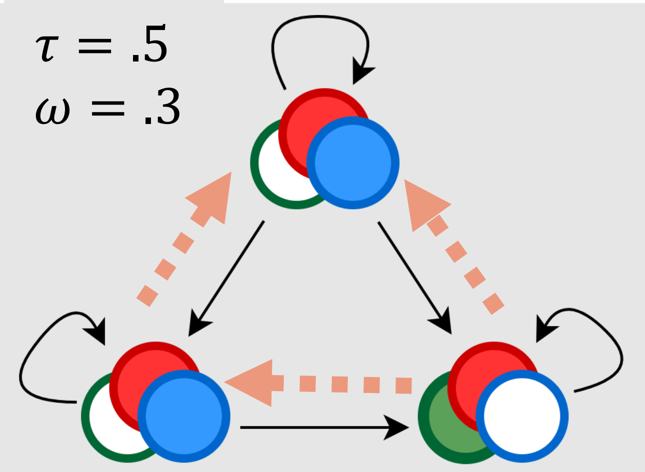
△ □

If **a diffusion step does not change who is similar to whom**, simultaneous = diffusion ↷ network.



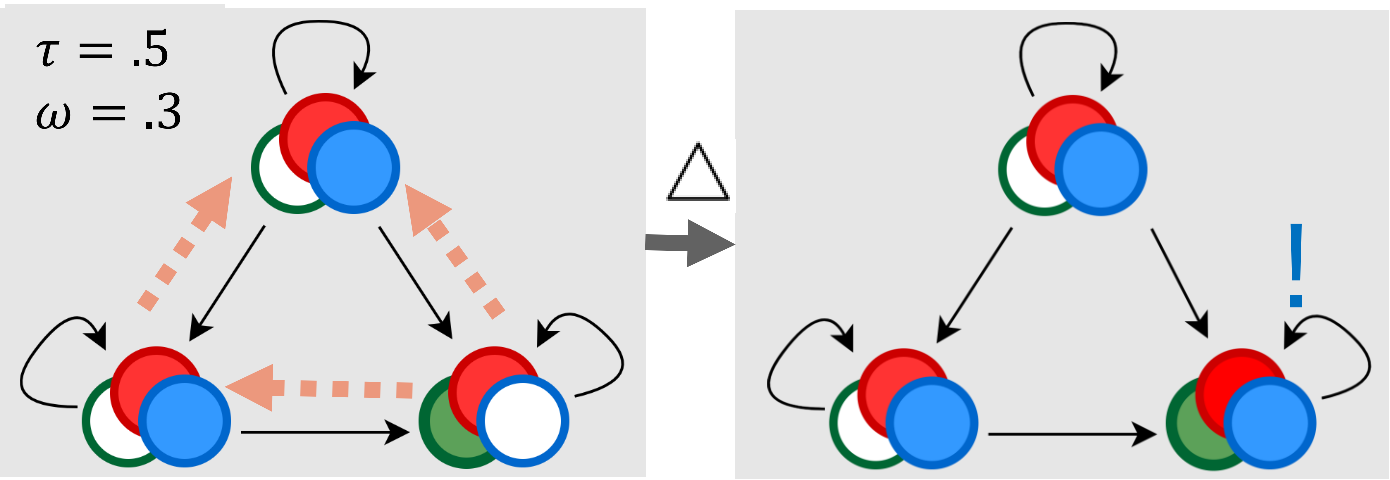
Replacing $S_{\{○\}}$

1. Characterisation of the class of models where the sequence ○ is replaceable.

Sequences that are equivalent to ○ on special classes of models:

△ □

If **a diffusion step does not change who is similar to whom**, simultaneous = diffusion $\curvearrowright$ network.



Replacing $S_{\{\bigcirc\}}$

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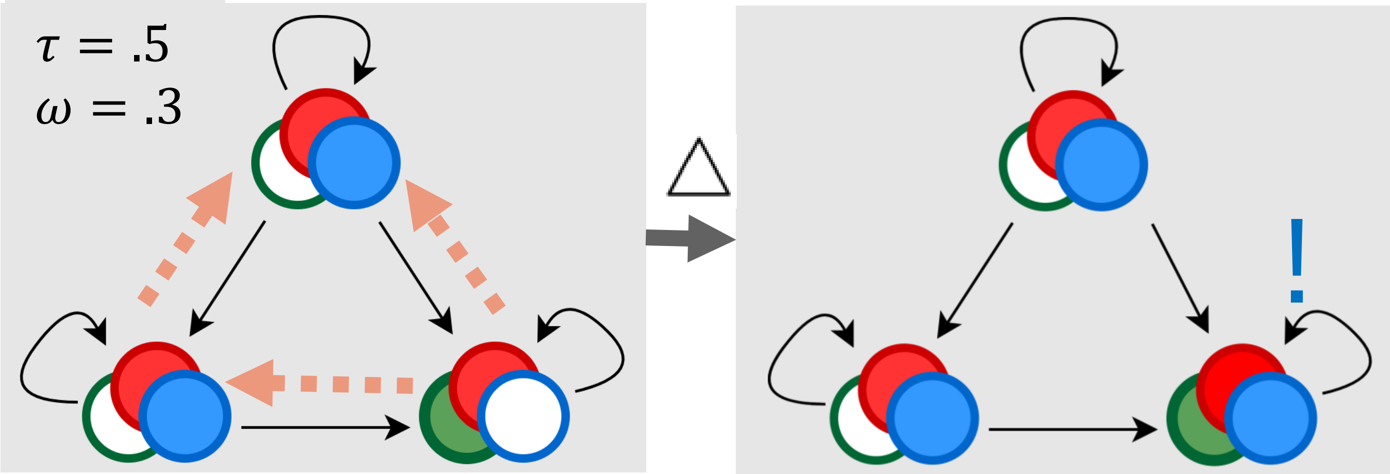
Sequences that are equivalent to ○ on special classes of models:

△ □

If **a diffusion step does not change who is similar to whom**, simultaneous = diffusion ↷ network.



All agents still similar!

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Sequences that are equivalent to ◯ on special classes of models:

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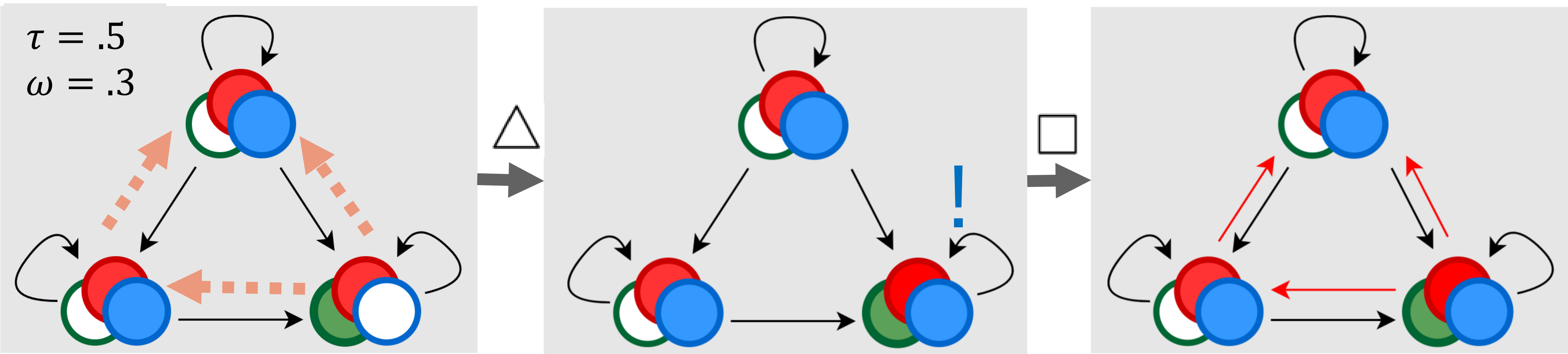

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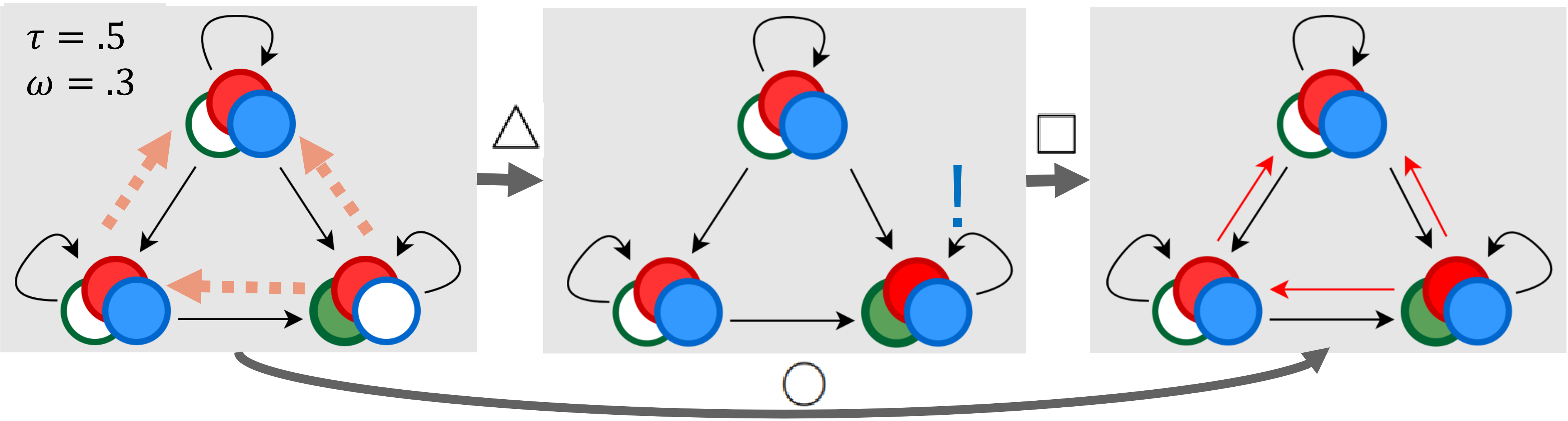
If **a diffusion step does not change who is similar to whom**, simultaneous = diffusion ↷ network.



Replacing $S_{\{\bigcirc\}}$

1. Characterisation of the class of models where the sequence $\circlearrowleft$ is replaceable.

Sequences that are equivalent to $\bigcirc$ on special classes of models:

$\triangle$ $\square$ $\triangle\square$

If **new links do not prevent one from later adopting features they would have adopted had those links not existed:**

simultaneous = network $\curvearrowright n$ diffusion steps ($\odot \equiv \boxdot\triangle^{n}$).

Replacing $S_{\{\bigcirc\}}$

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If **new links do not prevent one from later adopting features they would have adopted had those links not existed:**

simultaneous = network $\curvearrowright$ n diffusion steps ($\odot \equiv \boxdot\triangle^n$).

$\bigcirc$ is equivalent to $\square\triangle^n$ iff $M \models \bigwedge_{a\in A}\bigwedge_{f\in F}(\neg f_a \rightarrow (f^{\tau}_{N(a)} \leftrightarrow \bigvee_{0\leq i\leq n-1}\square\triangle^i f^{\tau}_{N(a)}))$

Replacing $S_{\{○\}}$

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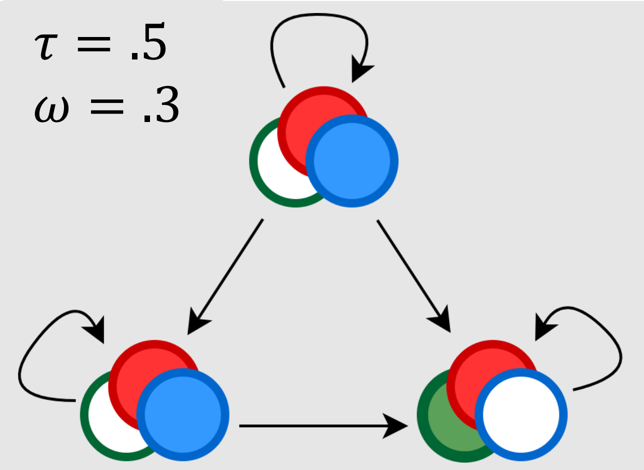
Sequences that are equivalent to ○ on special classes of models:

△ □ △□

If **new links do not prevent one from later adopting features they would have adopted had those links not existed,** simultaneous = network $\curvearrowright n$ diffusion steps.



Replacing $S_{\{\bigcirc\}}$

1. Characterisation of the class of models where the sequence ◯ is replaceable.

Sequences that are equivalent to ◯ on special classes of models:

△ □ △□

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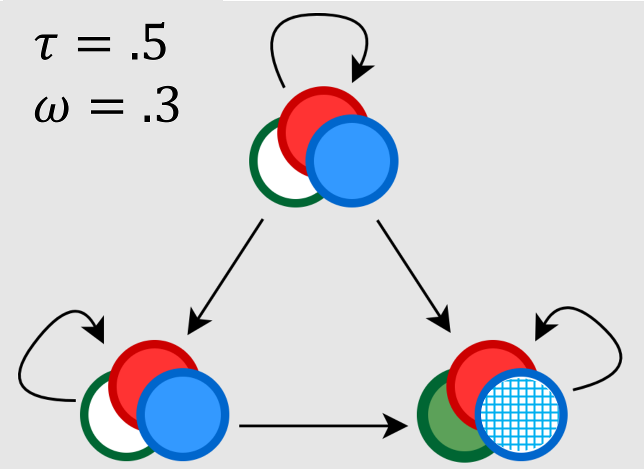


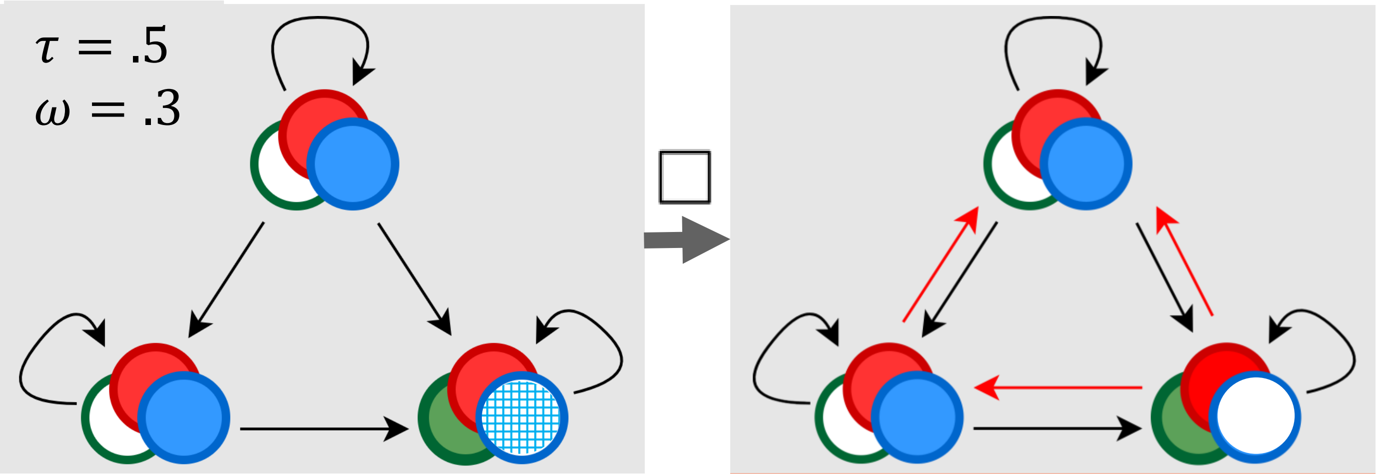
Replacing $S_{\{\bigcirc\}}$

1. Characterisation of the class of models where the sequence ↻ is replaceable.

Sequences that are equivalent to ◯ on special classes of models:

△ □ △□

If **new links do not prevent one from later adopting features they would have adopted had those links not existed,** simultaneous = network ↷ n diffusion steps.



Bottom left has the same neighbours!

Replacing $S_{\{○\}}$

○

1. Characterisation of the class of models where the sequence ○ is replaceable.

Sequences that are equivalent to ○ on special classes of models:

△ □ △□

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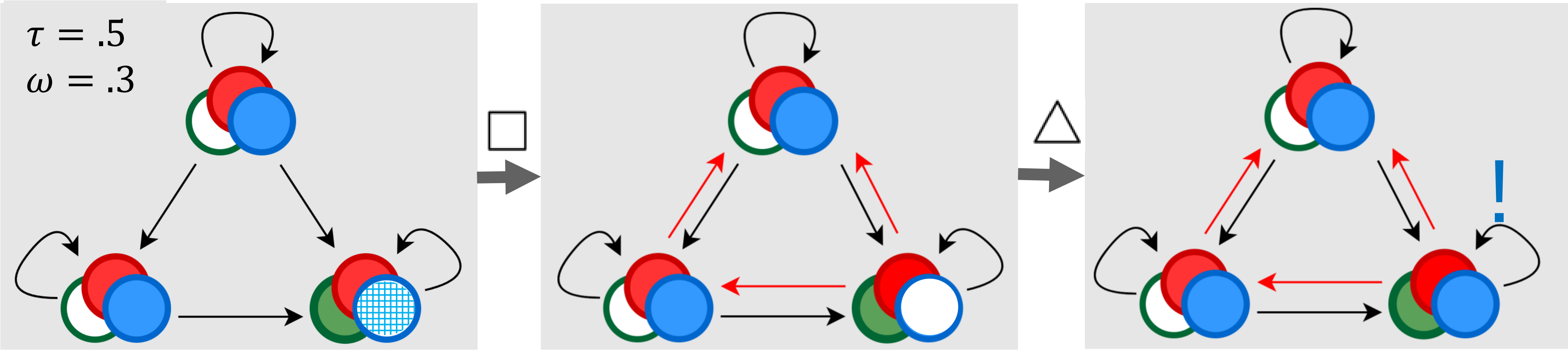

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Sequences that are equivalent to ○ on special classes of models:

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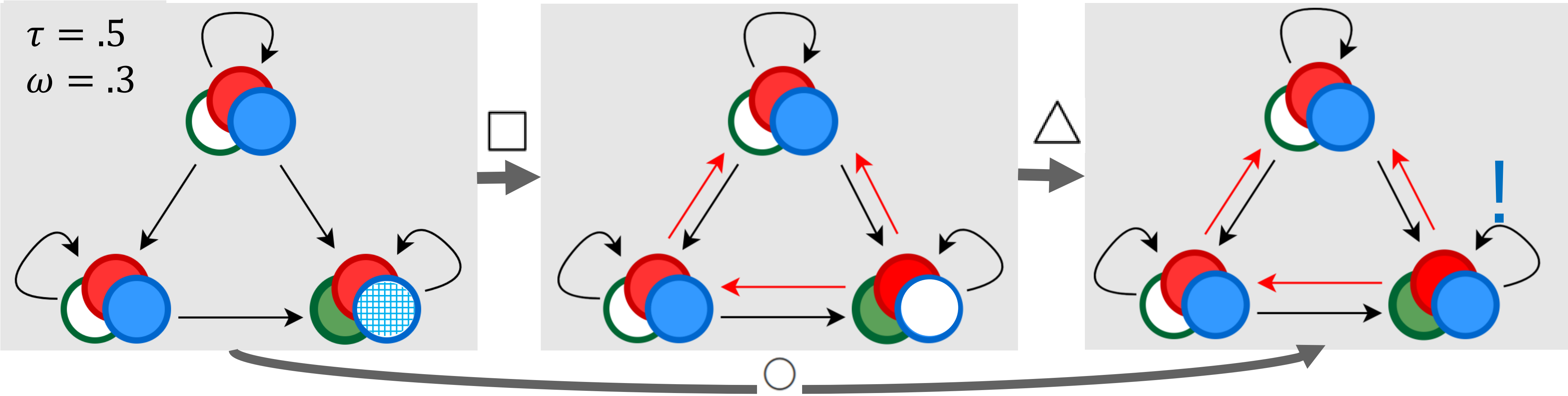

Replacing $S_{\{\bigcirc\}}$

1. Characterisation of the class of models where the sequence $\bigcirc$ is replaceable.

Sequences that are equivalent to $\bigcirc$ on special classes of models:

$\triangle$ $\square$ $\triangle\square$ $\square\triangle^n$

Replacing $S_{\{\bigcirc\}}$

1. Characterisation of the class of models where the sequence $\bigcirc$ is replaceable.

Sequences that are equivalent to $\bigcirc$ on special classes of models:

$\triangle$ $\qquad$ $\Box$ $\qquad$ $\triangle\Box$ $\qquad$ $\Box\triangle^n$

Any sequence equivalent to $\bigcirc$ on a model is equivalent to one of the sequences above.

Replacing $S_{\{\bigcirc\}}$

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Sequences that are equivalent to $\bigcirc$ on special classes of models:

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Any sequence equivalent to $\bigcirc$ on a model is equivalent to one of the sequences above.

The disjunction of equivalence formulas characterizes replaceability of $\bigcirc$ in a model.

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2. Sufficient condition for the replaceability of $S_{\{\bigcirc\}}$.

Replacing $S_{\{\bigcirc\}}$

1. Characterisation of the class of models where the sequence $\bigcirc$ is replaceable.

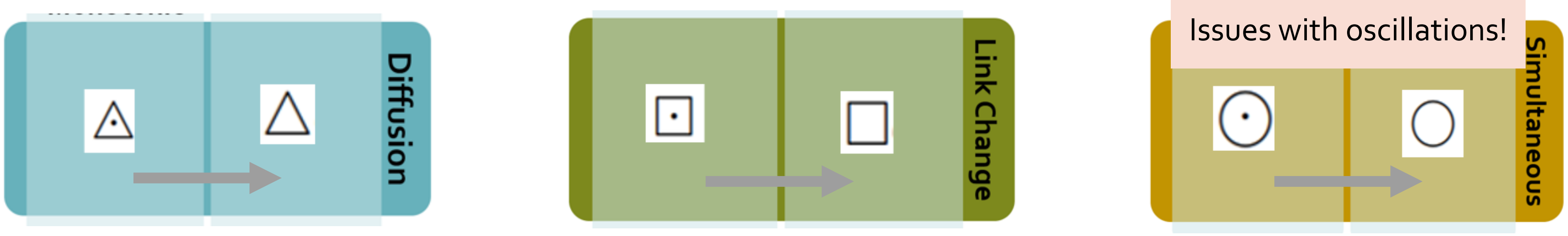
The disjunction of equivalence formulas characterizes replaceability of $\bigcirc$ in a model.

2. Sufficient condition for the replaceability of $S_{\{\bigcirc\}}$.

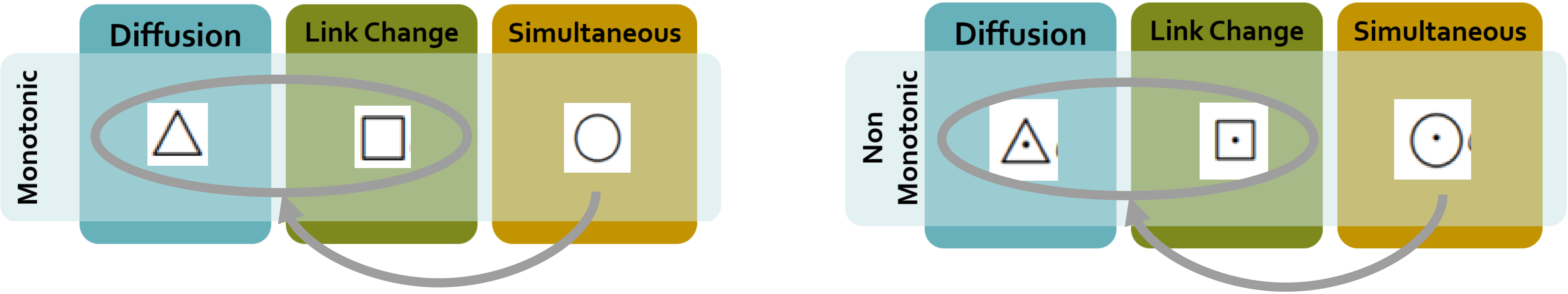
Obtained by requiring that $\bigcirc$ be replaceable after every update.

Replaceability on special model classes

1. Replaceability of non-monotonic changes by the corresponding monotonic changes.



2. Replaceability of simultaneous changes by the corresponding non-simultaneous changes.



Comparing Social Network Dynamics: **Summary**

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1. Designed a logic for diffusion and link changes in social network.

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2. Compared dynamics via replaceability

2.1 General irreplaceability result

Q	Can different sets capture different dynamics?	In general, yes.
Q	Can we reduce sets of operations to others?	Not always.

Comparing Social Network Dynamics: **Summary**

1. Designed a logic for diffusion and link changes in social network.

2. Compared dynamics via replaceability

2.1 General irreplaceability result

Q	Can different sets capture different dynamics?	In general, yes.
Q	Can we reduce sets of operations to others?	Not always.

2.2 Replaceability on special classes of models

Q	Can you sometimes replace a sequence set? When?	Focus on simultaneous changes.

Comparing Social Network Dynamics: **Further Work**

Further Work	**How frequently are operations replaceable?**
Further Work	**What about more complex type of updates?**
Further Work	**What happens if we would add anti-monotonic updates?**
Further Work	**What social configurations can be reached by different types of updates?**

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Image from https://pixabay.com/vectors/social-media-connections-networking-3846597/

university of groningen

Comparing Social Network Dynamics

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